Engineering Geology 168 (2014) 120–128

Contents lists available at ScienceDirect

Engineering Geology

journal homepage: www.elsevier.com/locate/enggeo

Slope reliability analysis considering spatially variable shear strength parameters using a non-intrusive stochastic finite element method

Shui-Hua Jiang [a], Dian-Qing Li [a,*], Li-Min Zhang [b], Chuang-Bing Zhou [a]

[a] State Key Laboratory of Water Resources and Hydropower Engineering Science, Key Laboratory of Rock Mechanics in Hydraulic Structural Engineering (Ministry of Education), Wuhan University, 8 Donghu South Road, Wuhan 430072, PR China

[b] Department of Civil and Environmental Engineering, The Hong Kong University of Science and Technology, Clear Water Bay, Kowloon, Hong Kong

ARTICLE INFO

Article history:
Received 31 July 2013
Received in revised form 31 October 2013
Accepted 10 November 2013
Available online 16 November 2013

Keywords:
Slopes
Shear strength
Spatial variability
Random field
Reliability
Stochastic finite element method

ABSTRACT

This paper proposes a non-intrusive stochastic finite element method for slope reliability analysis considering spatially variable shear strength parameters. The two-dimensional spatial variation in the shear strength parameters is modeled by cross-correlated non-Gaussian random fields, which are discretized by the Karhunen–Loève expansion. The procedure for a non-intrusive stochastic finite element method is presented. Two illustrative examples are investigated to demonstrate the capacity and validity of the proposed method. The proposed non-intrusive stochastic finite element method does not require the user to modify existing deterministic finite element codes, which provides a practical tool for analyzing slope reliability problems that require complex finite element analysis. It can also produce satisfactory results for low failure risk corresponding to most practical cases. The non-intrusive stochastic finite element method can efficiently evaluate the slope reliability considering spatially variable shear strength parameters, which is much more efficient than the Latin hypercube sampling (LHS) method. Ignoring spatial variability of shear strength parameters will result in unconservative estimates of the probability of slope failure if the coefficients of variation of the shear strength parameters exceed a critical value or the factor of slope safety is relatively low. The critical coefficient of variation of shear strength parameters increases with the factor of slope safety.

1. Introduction

In recent years, the spatial variability of soil properties has received considerable attention in slope stability analysis. Many investigators have contributed to this subject (e.g., Griffiths and Fenton, 2004; Cho, 2007; Low et al., 2007; Srivastava and Sivakumar Babu, 2009; Cho, 2010; Srivastava et al., 2010; Griffiths et al., 2011; Wang et al., 2011; Cho, 2012; Ji et al., 2012; Li et al., 2013c; Zhu and Zhang, 2013). For example, Griffiths and Fenton (2004) studied the effect of spatial variability of undrained shear strength on the probability of slope failure using random finite element method. Cho (2007) investigated the effect of spatially variable soil properties on the slope stability using direct Monte Carlo simulations (MCS). Low et al. (2007) proposed a practical EXCEL procedure to analyze slope reliability in the presence of spatially varying shear strength parameters. Srivastava and Sivakumar Babu (2009) quantified the spatial variability of soil parameters using field test data and evaluated the reliability of a spatially varying cohesive–frictional soil slope. Cho (2010) investigated the effect of spatial variability of shear strength parameters accounting for the correlation between cohesion and friction angle on the slope reliability. Srivastava et al. (2010) investigated the effect of spatial variability of permeability parameter on steady state seepage flow and slope stability. Griffiths et al. (2011) performed a probabilistic analysis to explore the influence of spatial variation in the shear strength parameters on the reliability of infinite slopes. Wang et al. (2011) developed a subset simulation-based reliability approach for slope stability analysis considering spatially variable undrained shear strength. Ji et al. (2012) adopted the First Order Reliability Method (FORM) coupled with a deterministic slope stability analysis to search the critical slip surface when the spatial variability in the shear strength parameters is considered.

In the majority of these studies, the traditional limit equilibrium method (LEM) is used to perform deterministic slope stability analyses. Then, the LEM is combined with random field theory for slope reliability analysis considering spatially variable soil properties. Thereafter, Monte Carlo Simulation is used to evaluate the probability of slope failure. A potential pitfall of the LEM is that some assumptions relating to the shape or location of the critical failure mechanism have to be made. Also, it does not account for the stress–strain behavior of the soil. Additionally, the spatial variability of soil properties cannot be considered realistically with the LEM-based methods, unless the shape of the slip surface is non-circular (Tabarroki et al., 2013). Fortunately, finite element based methods provide solutions to overcome the aforementioned

* Corresponding author at: State Key Laboratory of Water Resources and Hydropower Engineering Science, Wuhan University, 8 Donghu South Road, Wuhan 430072, PR China. Tel.: +86 27 6877 2496; fax: +86 27 6877 4295.
E-mail address: dianqing@whu.edu.cn (D.-Q. Li).

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http://dx.doi.org/10.1016/j.enggeo.2013.11.006

shortcomings underlying the traditional LEM (Farias and Naylor, 1998; Griffiths and Fenton, 2004). As for the slope reliability evaluation, although the direct MCS is simple and suitable for evaluating the probability of slope failure in the presence of spatially variable shear strength parameters, the time and resources required for the MCS could be prohibitive because a substantial number of finite element model runs are needed to obtain reliability results with a sufficient accuracy. The resultant computational efforts are most pronounced at relatively small probability levels or when complex finite element analyses are needed for slope stability analysis. Traditional stochastic finite element methods require significant modification of existing deterministic numerical codes, and become nearly impossible for most engineers with no access to the source codes of commercial software packages (Ghanem and Spanos, 2003; Stefanou, 2009). Therefore, it is necessary to explore more efficient methods for slope reliability analysis, which considers spatially variable shear strength parameters and requires complex finite element analysis for determining the factor of safety.

The objective of this paper is to propose a non-intrusive stochastic finite element method for slope reliability analysis considering spatially variable shear strength parameters. To achieve this goal, this article is organized as follows. In Section 2, the two-dimensional (2-D) spatial variation of the shear strength parameters is modeled by cross-correlated non-Gaussian random fields, which are discretized by the Karhunen–Loève (KL) expansion. In Section 3, the procedure of a non-intrusive stochastic finite element method is presented. Two examples of slope reliability analysis are investigated to demonstrate the capacity and validity of the proposed method in Section 4.

2. Random field modeling of soil property

2.1. Spatial variability of soil property

A Gaussian random field is completely defined by its mean $\mu(x)$, standard deviation $\sigma(x)$, and autocorrelation function $\rho(x_1, x_2)$. The autocorrelation function is an important physical quantity for characterizing the spatial correlation of soil properties (Vanmarcke, 2010). In this study, a squared exponential 2-D autocorrelation function is adopted with different autocorrelation distances in the horizontal and vertical directions as follows:

$$\rho[(x_1,y_1),(x_2,y_2)] = \exp\left(-\left[\left(\frac{|x_1-x_2|}{l_h}\right)^2 + \left(\frac{|y_1-y_2|}{l_v}\right)^2\right]\right) \tag{1}$$

where (x_1, y_1) and (x_2, y_2) are the coordinates of two arbitrary points in a 2-D space; and l_h and l_v are the autocorrelation distances in the horizontal and vertical directions, respectively.

2.2. Karhunen–Loève (KL) expansion

Several methods such as the midpoint method (Der Kiureghian and Ke, 1988), the local average subdivision (LAS) method (Vanmarcke, 2010), the shape function method (Liu et al., 1986) and the KL expansion (Phoon et al., 2002) can be used to discretize the random field. Since the KL expansion requires the minimum number of random variables for a prescribed level of accuracy, it is employed to discretize the 2-D anisotropic random fields of shear strength parameters. To facilitate the understanding of the proposed non-intrusive stochastic finite element method, the KL expansion is introduced briefly in the following.

A random field $\mathbf{H}(x, \theta)$ is a collection of random variables associated with a continuous index $x \in \Omega \subseteq R^n$, where Ω is an open set of R^n describing the system geometry and $\theta \in \Theta$ is the coordinate in the outcome space. Discretization of a random field using the KL expansion is based on the spectral decomposition of its autocorrelation function $\rho(x_1, x_2)$. Generally, the autocorrelation function is bounded, symmetric and positive definite. Hence, the discretization of a random field is defined by the eigenvalue problem of the homogenous Fredholm integral equation as follows:

$$\int_\Omega \rho(x_1,x_2) f_i(x_2) dx_2 = \lambda_i f_i(x_1) \tag{2}$$

where x_1 and x_2 denote the coordinates of two points; $f_i(\cdot)$ and λ_i are the eigenfunctions and eigenvalues of the 1-D autocorrelation function $\rho(x_1, x_2)$, respectively. Then, the eigenmodes of the separable multi-dimensional autocorrelation function are calculated by multiplying with the eigenmodes obtained from Eq. (2) (e.g., Huang, 2001).

The eigenvalue problem of the Fredholm integral equation in Eq. (2) is often solved numerically due to its complexity. The wavelet-Galerkin technique is adopted herein to solve the above eigenvalue problem. More details are given by Phoon et al. (2002). The series expansion of a 2-D random field $\mathbf{H}_i(x, y)$ is expressed as

$$\boldsymbol{H}_i(x,y) = \mu_i + \sum_{j=1}^{\infty} \sigma_i \sqrt{\lambda_j} f_j(x,y)\xi_{i,j}, \quad x,y \in \Omega \tag{3}$$

where $\xi_{i,j}$ is a set of orthogonal random coefficients (uncorrelated random variables with zero mean and unit variance). The series expansion in Eq. (3), referred to as the KL expansion, provides a second-moment characterization in terms of uncorrelated random variables and deterministic orthogonal functions. It is known to converge in the mean square sense for any distribution of $\mathbf{H}_i(x, y)$ (e.g., Vořechovský, 2008). For practical implementation, the series is approximated by a finite number of terms in Eq. (3):

$$\widetilde{\boldsymbol{H}}_i(x,y) = \mu_i + \sum_{j=1}^{n} \sigma_i \sqrt{\lambda_j} f_j(x,y)\xi_{i,j}, \quad x,y \in \Omega \tag{4}$$

where n is the number of KL expansion terms to be retained, which highly depends on the desired accuracy and the autocorrelation function of the random field. Small values of the autocorrelation distances will lead to a significant increase in the number of the eigenmodes, n. Several studies (Huang, 2001; Laloy et al., 2013) took the ratio of the expected energy, ε, as a measure of the accuracy of the truncated series, which is defined as

$$\begin{aligned}\varepsilon &= \int_\Omega E\left(\widetilde{\boldsymbol{H}}_i(x,y)-\mu_i\right)^2 dxdy \Big/ \int_\Omega E(\boldsymbol{H}_i(x,y)-\mu_i)^2 dxdy \\ &= \sum_{i=1}^{n}\lambda_i \Big/ \sum_{i=1}^{\infty}\lambda_i\end{aligned} \tag{5}$$

where the eigenvalues λ_i are sorted in a descending order. A large ε always indicates a high accuracy of the truncated series.

2.3. Cross-correlated non-Gaussian random fields

In geotechnical engineering practice, very often more than one geotechnical parameter needs to be modeled by random fields. Furthermore, the geotechnical engineering literature is replete with cross-correlations between two geotechnical parameters. For example, the two curve-fitting parameters underlying load–displacement curve of piles are negatively correlated (Li et al., 2013b). The cohesion and friction angle, often used for slope reliability analysis, are considered to be negatively correlated (e.g., Lumb, 1970; Wolff, 1985; Cho, 2010; Tang et al., 2013). In this case, the cross-correlated random fields need to be handled. Following Cho (2010), it is assumed that all fields simulated over a region Ω share an identical autocorrelation function over Ω, and the cross-correlation structure between each pair of simulated fields is simply defined by a cross-correlation coefficient. It can ensure that the target random fields respect the correlation structure within each field (Vořechovský, 2008). The rationale underlying these assumptions has been explained by Fenton and Griffiths (2003). Under these