Engineering Geology 179 (2014) 1–9

Contents lists available at ScienceDirect

Engineering Geology

journal homepage: www.elsevier.com/locate/enggeo

Estimation of soil–water characteristic curve and relative permeability for granular soils with different initial dry densities

Wan-Huan Zhou, Ka-Veng Yuen *, Fang Tan

Department of Civil and Environmental Engineering, Faculty of Science and Technology, University of Macau, Macau, China

ARTICLE INFO

Article history:
Received 29 September 2013
Received in revised form 10 June 2014
Accepted 14 June 2014
Available online 27 June 2014

Keywords:
Bayesian method
Initial dry density
SWCC
Relative permeability function
Uncertainty
Unsaturated soil

ABSTRACT

This paper presents a new estimation model to consider the effect of initial dry density on the soil–water characteristic curve (SWCC) of granular soils. The proposed method is based on the Fredlund and Xing equation without increasing the number of adjustable parameters. The experimental data from a soil database is partitioned into two groups for fitting and verification of the proposed method. From the verification results, it can be seen that the proposed method provides significantly better estimation of SWCC for the granular soils with different initial dry densities than the Fredlund and Xing equation. From the comparison and statistical analysis, it is found that the estimations by the present method exhibit higher correlations with experimental data than the Tarantino's model for the granular soil samples. The uncertainty of measurement data is analyzed by using the Bayesian method and applied in the estimation of the relative permeability function (k_r). Different levels of confidence intervals of SWCC are obtained by the Bayesian approach. It is found that the proposed method can better predict the k_r values with smaller uncertainty for the soil with different initial dry densities than the Fredlund and Xing equation. The new method is readily adopted for engineers to quickly estimate the SWCC and relative permeability of granular soil under different initial dry densities.

1. Introduction

The measurement of soil parameters for unsaturated soils requires complicated laboratory tests and it is always time consuming. To avoid cumbersome laboratory work, the soil–water characteristic curve (SWCC) is often used to estimate various unsaturated soil property functions, such as shear strength function, permeability function, and thermal property function. At present, many methods or soil property indexes have been adopted to estimate SWCC. The statistical correlations between water potential at certain matric suction and soil texture have been developed (Gupta and Larson, 1979; Saxton et al., 1986). Some researchers (Ahuja et al., 1985; Chin et al., 2010) established empirical relations between soil properties and fitting parameters of the SWCC equation. Some researchers used a physics-based conceptual model to estimate SWCC, such as the work by Arya and Paris (1981) and Fredlund et al. (1997) among others. This approach involves a conversion of the grain-size distribution into the pore-size distribution of a soil. Similarly, Fredlund and Xing (1994) proposed an SWCC equation based on the assumption that the shape of SWCC depends on the pore-size distribution of the soil. Leong and Rahardjo (1997) reviewed and evaluated five popular SWCC equations and found that the Fredlund and Xing (1994) equation gave the best fit among the others. Zhang and Chen (2005) extended the Fredlund and Xing (1994) model and the van Genuchten (1980) model to describe bimodal and multimodal SWCCs.

The influencing factors of SWCC have been studied by previous researchers. Zhou and Yu (2005) found that the initial water content and stress state were important influencing factors. It is well accepted that the initial dry density of soil has a significant influence on the SWCC. Yang et al. (2004) showed that the shape of SWCC was similar to the grain-size distribution of the soil and was affected by the dry density of the soil. Similarly, Rajkai et al. (1996) studied the estimation of water retention characteristic from bulk density and particle size distribution parameters. Vanapalli et al. (1999) revealed that the effect of initial water content and porosity on SWCC tends to diminish at high suction level. Gallipoli et al. (2003) suggested an improved form of the van Genuchten (1980) SWCC model to account for the influence of void ratio. Tarantino (2009) proposed an SWCC equation for deformable soils based on an empirical power function of the water ratio. Gallage and Uchimura (2010) investigated the effect of dry density and grain size distribution on SWCC's parameters of sandy soils. Sheng and Zhou (2011) studied the effect of deformation on the soil–water retention properties and found the influence of initial void ratio on the air-entry value and the slope of SWCC. Therefore, the effect of initial dry density of soil should be sufficiently considered in estimating the SWCC of granular soils.

* Corresponding author at: Department of Civil and Environmental Engineering, Faculty of Science and Technology, University of Macau, Av. Padre Tomás Pereira, Taipa, Macau, China. Tel.: +853 8397 4960; fax: +853 28838314.
E-mail address: kvyuen@umac.mo (K.-V. Yuen).
Personal Homepage: http://www.sftw.umac.mo/kvyuen (K.-V. Yuen).

http://dx.doi.org/10.1016/j.enggeo.2014.06.013
0013-7952/

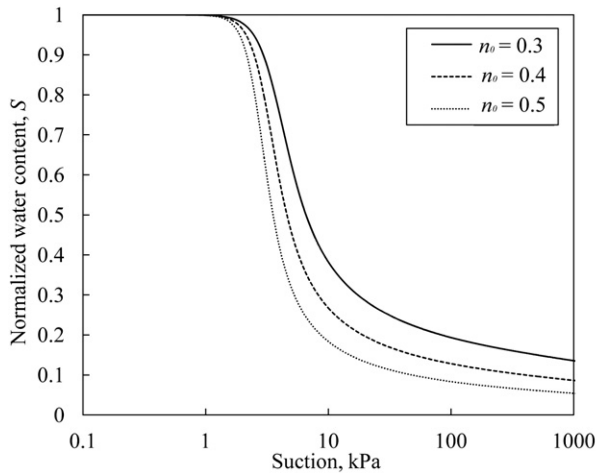

Fig. 1. The SWCCs for the same soils under different initial porosities ($a^* = 1, k^* = 15, m^* = 2$).

Generally speaking, the measured data are firstly needed to obtain the optimal curve-fitting parameters in the SWCC models. However, it should be noted that the measured data of SWCC and the fitting parameters always exhibit high level of uncertainty due to the complicated unmodeled influencing factors. For the geotechnical uncertainties, Phoon and Kulhawy (1999a, 1999b) summarized its three primary sources, *i.e.* inherent variability, measurement error and transformation uncertainty. The uncertainty of empirical models can be considered by regarding the curve-fitting parameters as a random vector (Phoon et al., 2010; Wang et al., 2011; Chiu et al., 2012). Some probabilistic measures have also been adopted to consider the uncertainties of empirical models or parameters in other civil or geotechnical engineering problems (Phoon and Kulhawy, 2003; Phoon et al., 2003; Zhang et al., 2004; Ching et al., 2009; Yan et al., 2009; Wang et al., 2010; Yuen, 2010b; Yuen and Kuok, 2011; Yuen and Mu, 2011; Ching et al., 2012; Juang et al., 2012; Zhou et al., 2013). Recently, Zhai and Rahardjo (2013) developed the equations for the confidence limits of the best-fit SWCC to quantity the uncertainties in the associated fitting parameters.

For the same granular soil under different initial dry densities, their SWCCs are different. In this paper, considering the effect of initial dry density, we propose a new estimation method based on the Fredlund and Xing (1994) equation. The data from the Unsaturated Soil Hydraulic Database (UNSODA, Leij et al., 1996) are used for fitting and verification of the proposed method. The estimation results by the Fredlund and Xing (1994) model and Tarantino (2009) model will also be presented for comparison. It is found that the proposed method provides better predictions for the granular soils with different initial dry densities than the Fredlund and Xing model. Both the Tarantino's model and the present model consider the effect of the initial void ratio on the SWCC. From the comparison and statistical analysis, it is found that the estimations by the present method exhibit higher correlations with experimental data than the Tarantino's model for the granular soil samples. In addition, the uncertainty of the fitting parameters in the SWCC equations was analyzed by using the Bayesian method, and then the SWCC and relative permeability function (k_r) with different percentiles were predicted. The study shows that the estimated values of k_r by the proposed method are associated with smaller uncertainty than those by the Fredlund and Xing model for the soils with different initial dry densities.

2. Estimation of SWCCs with different initial dry densities

2.1. Fredlund and Xing (1994) equation for SWCC

Based on the assumption that the shape of the SWCC depends on the pore-size distribution of the soil, Fredlund and Xing (1994) proposed a three-parameter equation for the SWCC, with flexibility to fit a wide range of soils:

$$\theta = \frac{\theta_s}{\{\ln[\exp(1) + (\varphi/a)^k]\}^m}\left[1 - \frac{\ln(1+\varphi/\varphi_r)}{\ln(1+10^6/\varphi_r)}\right] \quad (1)$$

where θ_s is the saturated volumetric water content; a is the fitting parameter related to the air-entry value for the soil; k is the fitting parameter related to the maximum slope of the curve; m is the fitting parameter related to the curvature of the curve; φ is the soil suction; φ_r represents the soil suction related to the residual volumetric water content. In order to reduce the complexity of the equation, it was suggested that the residual suction φ_r takes the value of 3000 kPa, regardless of the soil types (Sillers and Fredlund, 2001). In addition, the normalized volumetric water content, *i.e.* $S = \frac{\theta}{\theta_s}$, is introduced and Eq. (1) can be re-written as:

$$S = \frac{1}{\{\ln[\exp(1) + (\varphi/a)^k]\}^m}\left[1 - \frac{\ln(1+\varphi/\varphi_r)}{\ln(1+10^6/\varphi_r)}\right]. \quad (2)$$

The fitting parameters, *i.e.* a, k and m, can be obtained by nonlinear least-square methods. Although the Fredlund and Xing equation is capable to describe the SWCC by fitting the measured data, it cannot distinguish the SWCCs for the same soil under different initial dry densities because the same set of fitting parameters (a, k, m) is used in the Fredlund and Xing equation to describe their SWCCs. In other words, for the same soils with different initial dry densities, their SWCCs (Eq. (2)) are identical. This induces a higher level of prediction uncertainty in the estimation of unsaturated soil properties as the soil initial dry density has a significant influence on the SWCC, permeability, and shear strength, *etc.*

2.2. Effect of initial dry density and proposed estimation method

Soil is a porous medium with different particle sizes and complex texture. Granular soils at different compaction states are associated with different dry densities and porosities. The water in the porous

Table 1
Soil properties and texture contents.

Soil code	Soil type	n_0	θ_s	ρ_d (g/cm^3)	k_s (m/s)	Sand (%) 0.05–2 mm	Silt (%) 0.002–0.05 mm	Clay (%) <0.002 mm
1460	Berlin coarse sand	0.297	0.261	1.85	2.91e−5	98	0.8	1.2
1461	Berlin coarse sand	0.373	0.362	1.65	2.31e−4	99	0.7	0.3
1462	Berlin medium sand	0.430	0.315	1.50	1.16e−4	97	1.4	1.6
1463	Berlin medium sand	0.399	0.388	1.58	8.01e−5	97	1.9	1.1
1464	Berlin medium sand	0.365	0.357	1.67	2.31e−5	93	4.4	2.6
1140	Wagram sand	0.428[a]	0.385	1.56	2.05e−5	92	5	3
1141	Wagram sand	0.336[a]	0.302	1.70	4.47e−6	90	8	2
1142	Wagram sand	0.272[a]	0.245	1.80	1.55e−6	87	11	2

[a] The initial porosity is obtained through the saturated volumetric water content divided by 0.9 (Pachepsky et al., 1999).