Engineering Geology 179 (2014) 90–101

Contents lists available at ScienceDirect

Engineering Geology

journal homepage: www.elsevier.com/locate/enggeo

Unified continuum/discontinuum modeling framework for slope stability assessment

Wenbo Zheng [a,b], Xiaoying Zhuang [b,c,*], Dwayne D. Tannant [a], Yongchang Cai [b], Samuel Nunoo [a]

[a] School of Engineering, The University of British Columbia, Kelowna, British Columbia, V1V 1V7, Canada
[b] Key Laboratory of Geotechnical and Underground Engineering of Ministry of Education, Department of Geotechnical Engineering, College of Civil Engineering, Tongji University, 1239 Siping Road, Shanghai, 200092, China
[c] School of Engineering and Computing Sciences, Durham University, South Road, DH1 3LE, Durham, UK

ARTICLE INFO

Article history:
Received 29 March 2014
Received in revised form 9 May 2014
Accepted 14 June 2014
Available online 26 June 2014

Keywords:
Rock/soil slope
Stability analysis
Numerical manifold method
Graph theory
Discontinuum analysis

ABSTRACT

Current dominant methods for slope stability analysis are the limit equilibrium method and the strength reduction method. Both methods are based on the limit equilibrium conditions. However, the limit equilibrium methods are limited to the rigid body assumption, while the strength reduction method is computationally expensive and has convergence issues due to the non-linear iterative computations. In this paper, we propose a current stress-based search algorithm to directly obtain the critical slip surface and the safety factor. The numerical manifold method, which unifies the continuum and discontinuum analysis problems, is used for stress analysis to obtain the stress distribution of soil slopes or rock slopes cut by joints. Based on the stress results obtained, a graph theory is used to convert the solution of the critical slip surface to a shortest path problem, which can be directly solved by the Bellman–Ford algorithm. The proposed method couples the numerical manifold method and the graph theory allowing for stability analyses of both rock and soil slopes within the same framework. The method completely removes the computational effort needed for iterations in the strength reduction method as well as eliminating the rigid body assumptions in the limit equilibrium method.

1. Introduction

A wide range of methods have been proposed to assess stability of soil or rock slope such as those discussed by Fellenius (1936), Abramson et al. (2001), Duncan and Wright (2005). Among these methods, the limit equilibrium methods (LEM) are the most widely used due to their simple formulation. In the LEMs, the potential sliding body is discretized into a finite number of rigid slices. Different assumptions can be introduced concerning the equilibrium conditions and the inter-slice forces to make the slope stability problem statically determinate. In the family of LEMs, some well-known methods include those by Bishop (1955; Bishop modified), Morgenstern–Price (1965), Janbu (1968) and Sarma (1973). In particular, the Sarma method has been widely applied in rock slope engineering.

In the past three decades, many researchers such as Chen and Morgenstern (1983), Lam and Fredlund (1993) and Zhu et al. (2003) have tried to incorporate various types of LEM into a generalized framework. The LEMs have rigorous theoretical derivations and can provide the critical slip surface (CSS) directly. However, they are limited by the rigid body assumption and do not consider the constitutive models describing the stress–strain relation in the geomaterial. Therefore, the loading path cannot be modeled (Ching and Fredlund, 1983; Zhu et al., 2003).

To overcome the limitations of the LEMs, many numerical methods have been developed for slope stability analysis that are capable of analyzing more complicated loading conditions, geometries, material models, slope geometries and multi-field problems. According to the assumptions of kinematics and material models, numerical methods at present can be mainly classified into two categories, namely continuum-based methods and discontinuum-based methods (Jing and Hudson, 2002). Continuum-based methods are capable of simulating slopes with few joints where kinematic conditions do not control the slope behavior. However, continuum-based methods such as the finite element method (FEM) have been shown ineffective or cumbersome in modeling joint propagation since the mesh needs to be refined to cope with the evolving geometries (Zhuang and Augarde, 2010; Zhu et al., 2011). Singular elements in discontinuities might result in unexpected erroneous results and require robust and advanced analysis software to ensure numerical simulation with convergence. Element-free methods have greatly improved the ability to handle essential boundary conditions and the numerical integration (Zhuang et al., 2011, 2012; Zhuang and Cai, 2014), but they are still problematic when dealing with multiple fractures (Ma et al., 2010). Discontinuum-based methods such as the discrete element method (Cundall and Strack, 1979) and the discontinuous deformation analysis (Shi and Goodman, 1989) are

* Corresponding author.
E-mail address: xiaoying.zhuang@gmail.com (X. Zhuang).

http://dx.doi.org/10.1016/j.enggeo.2014.06.014
0013-7952/

capable of simulating discontinuous problems like jointed rock slopes. However, the efficiency of calculations is low and the parameters needed for calculations are difficult to determine. The numerical manifold method (NMM) proposed by Shi (1991) is a particular numerical method that unifies continuum analysis and discontinuum analysis. This method has attracted attention from many researchers during the past two decades (Chih and Haw, 1996; Sasaki et al., 1997; Li et al., 2005; Jiang et al., 2009; Cai et al., 2010; An et al., 2013; Cai et al., 2013). The NMM is particularly suitable for the stability analysis in both rock and soil slopes (Ning et al., 2011).

To address the above issues, this paper proposes an effective searching technique to assess slope stability in the context of NMM by using a graph theory. First, NMM is used to compute the stress state in the slope. Second, based on the stresses obtained, a search for the CSS with the SF is converted to a shortest path problem by appropriately defining vertexes and edges in the slope model. Finally, the shortest path problem is solved by the Bellman–Ford algorithm and the CSS is obtained directly. The combination of NMM and the graph theory removes the difficulties in iterative calculations and also unifies the analysis formulation for both rock and soil slopes.

2. Numerical manifold method of modeling discontinuities

Modeling of soil slopes without discontinuities in NMM is identical to that in the finite element method. For rock slopes with multiple discontinuities, the NMM uses dual covers to model slopes in a more natural way since the mathematical cover does not need to conform to the moving external and internal boundaries such as faults and joints. With a regular mathematical mesh, the inaccuracies and numerical instabilities associated with the distortion of element topology can be alleviated. Hence, another advantage of NMM arises when simulating fracture propagation without mesh refinement by updating its physical cover. However, this is beyond the scope of the present paper.

2.1. Basic formulation

In the NMM, a dual cover system is used, which comprises the mathematical covers and physical covers. In this paper, a triangular mesh illustrated in Fig. 1 is firstly used over an example problem domain denoted as Ω. Notably, the triangular mesh does not need to conform to the slope boundaries or the material discontinuities. A mathematical cover denoted as M_i, is defined as a triangular mesh that contains the mathematical node i where $i = 1$ to n_m and n_m is the total number of mathematical covers used to cover domain Ω. For example, the mathematical covers M_2 and M_9 are defined as the unions of elements connected to nodes 2 and 9, respectively, see Fig. 2. External boundaries and internal discontinuities divide each mathematical cover into different parts. Each part is termed as a physical cover. As shown in Fig. 3(a), M_2 is divided by the external boundary into P_2 while the domain outside the external boundary is not considered as a physical cover, see Fig. 3(a).

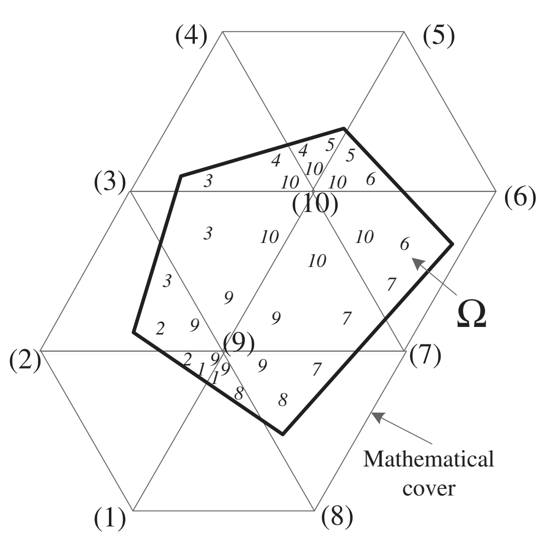

Fig. 1. Illustration of the concept of the NMM.

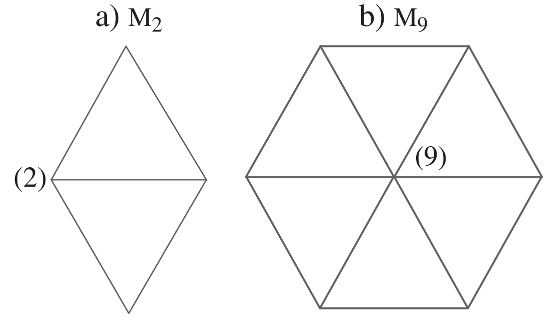

Fig. 2. Definition of a mathematical cover.

A manifold element is defined as the overlapping of the physical covers. In Fig. 3, the manifold element ME_{239} is generated as the intersection of the physical covers P_2, P_3 and P_9. The manifold elements are used to construct the local displacement approximation functions and to integrate the elemental stiffness matrix.

Over each mathematical cover M_i, a weight function $w_i(x,y)$ is defined that satisfies the following conditions

$$\begin{aligned} & w_i(x,y) \geq 0, (x,y) \in M \\ & w_i(x,y) = 0, (x,y) \notin M \\ & \sum_i^n w_i(x,y) \equiv 1, (x,y) \in \Omega. \end{aligned} \tag{1}$$

A local displacement approximation function $u_i(x,y)$ is defined on each physical cover P_i. Generally, polynomial functions are used to represent local displacement characteristics. High order polynomials can effectively improve the approximation accuracy. However, for simplicity, the local displacement approximation function is set as constant in this paper.

A global approximation $u^h(x,y)$ on each manifold element ME_j can be obtained by the summation of all the local approximation functions of related physical covers with their corresponding weight functions as

$$u^h(x,y) = \sum_i w_i(x,y) u_i(x,y), (x,y) \in ME_j. \tag{2}$$

The mathematical covers in the NMM do not need to be identical to the problem domain, while in the FEM, the mesh has to conform to the geometry of the problem. The FEM mesh can be viewed as a mathematical mesh that conforms exactly to the domain geometry and in this case the physical cover and the mathematical cover are identical. This is actually what has been done in many existing mesh-refinement techniques found in the FEM software. Like the FEM, the governing equations in the NMM are the equilibrium conditions, compatibility equations and constitutive equations, and the weak form can be derived based on the minimization total potential energy. The elemental stiffness matrix of each manifold element is constructed first. Subsequently, the element matrices are assembled into the global stiffness matrix. Different from the FEM, the weight functions in the NMM are independent of the geometries of the manifold elements.