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Technical Note

A simplified approach for modeling spatial variability of undrained shear strength in out-plane failure mode of earth embankment

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ABSTRACT

This paper presents a probabilistic study of the out-plane failure mode of long embankments by first-order reliability method (FORM) and limit equilibrium method (LEM). The uncertainty and longitudinal spatial variability of soils that are deemed the most important factors producing the 3-D failure modes are considered in the probabilistic analysis. The longitudinal (out-plane) spatial variation of undrained shear strength is modeled by both spatial-averaging approach and spatial-autocorrelation approach. Comparisons are made between the two methods. For a long earth embankment, the local 3-D failure with a probabilistic critical width is investigated.

1. Introduction

In geotechnical engineering, the spatial variability of natural soils is widely recognized and probabilistic analysis catering to this characteristic has been very active recently (Low and Tang, 1997; Cho and Park, 2010; Ji et al., 2012; Ching and Phoon, 2013; Jiang et al., 2014; Li et al., 2014). In the stability analysis of earth slopes, the spatial variability of shear strength parameters significantly influences the likely failure mode (Ji, 2012). Nevertheless, most previous studies by probabilistic analysis were limited to the framework of in-plane failure mode. A more rational study is to account for the out-plane spatial variability in a 3-D failure mode.

The 3-D slope stability can be analyzed using either the limit equilibrium method (LEM) of columns which involves various assumptions about the inter-column forces and the shape of sliding surface (Chen and Chameau, 1983; Lam and Fredlund, 1993; Huang et al., 2002; Cheng and Yip, 2007; Zheng, 2012; Zhou and Cheng, 2014), or the strength-reduction finite element method (SRFEM) (Ugai and Leshchinsky, 1995; Deng et al., 2007; Griffiths and Marquez, 2007). Extension of the deterministic 3-D stability analysis to probabilistic treatment has been seen in several studies (Vanmarcke, 1977b, 1980, 2011; Yücemen and Al-Homoud, 1990; Al-Homoud and Tanash, 2004; Griffiths et al., 2009; Hicks and Spencer, 2010). The pioneer work by Vanmarcke (1977b) showed that spatial variability of soils would significantly influence the 3-D failure mode. The random finite element method (RFEM) has recently been developed for probabilistic study of 3-D earth slopes (Griffiths et al., 2009; Hicks and Spencer, 2010). It is showed that the length of slope in out-plane direction affects the performance of 3-D stability. These probabilistic FEM stability analyses were all realized by MC simulations; they all yielded system failure probability and failed to reveal the local failure mode with a critical failure width, as rightly pointed out by Vanmarcke (2011).

This study aims at investigating the effect of out-plane spatial variability of soils on the 3-D failure mode using probabilistic analysis. The first-order reliability method (FORM) is employed.

2. Limit equilibrium stability analysis of an earth embankment

For earth embankments or levees, it is of great interest to investigate the likely failure mode along the longitudinal direction. For simplicity, a hypothetical model as shown in Fig. 1 is adopted to study the fundamental failure mechanism by LEM. The model assumes that the 3-D failure surface is cylindrical and extends along the longitudinal axis over a finite width b. The cylindrical failure mass has vertical cut-offs at two ends. Suppose the embankment has a uniform slope at 1.5H:1V, with a height H of 5 m and a foundation depth D of 2 m. For short-term stability analysis, the undrained shear strength c_u is 25 kPa and the total unit weight γ is 19 kN/m^3. As a starter, a plane strain stability analysis with search for the critical slip circle obtains a factor of safety F_s of 1.596.

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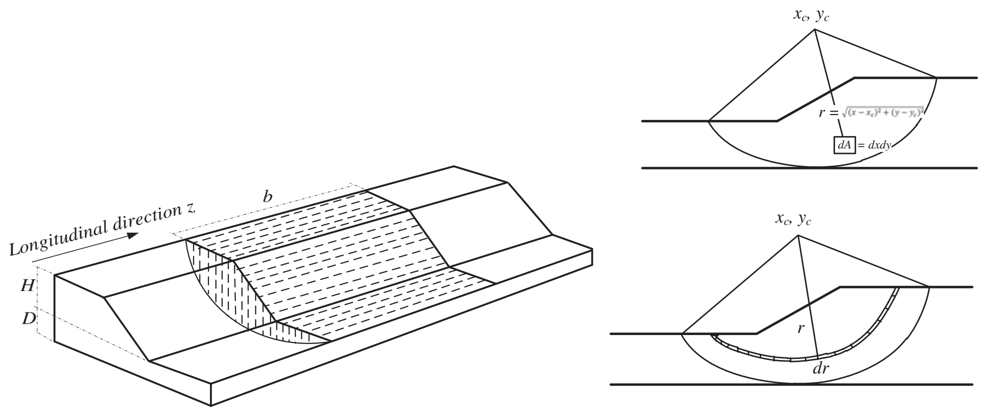

Fig. 1. A simplified model of failure mass of a long embankment.

To evaluate the 3-D failure mode, the F_{3D} which considers the end-section shear resistance is defined (Vanmarcke, 1977b, 1980):

$$F_{3D} = \frac{\sum M_R}{\sum M_O} = \frac{M_R b + 2M_{ER}}{M_O b} \tag{1}$$

where M_R and M_O are the resisting and overturning moments by plane strain analysis (moment per unit longitudinal length) with respect to x_c and y_c defining the critical slip circle, respectively, and M_{ER} is the resisting moment resulted from shear at end sections. Because only c_u contributes to the resisting moment, M_{ER} can be obtained by integrating the resisting moment of dA or $dxdy$ over the area of end section, as shown in Fig. 1.

$$M_{ER} = \iint_A c_u \sqrt{(x-x_c)^2 + (y-y_c)^2}\, dxdy \tag{2}$$

where x_c and y_c are respectively the rotation center coordinates defining the end-section. Also, the same value of M_{ER} can be obtained by integrating the resisting moment of dr. However, both the integration function and boundary condition are more complicated by doing so. Thus, Eq. (2) is recommended for the calculation of M_{ER} in this study.

From Eq. (1), it is obvious that F_{3D} consistently decreases with increase of b because the end-section shear resistance M_{ER} remains constant. It implies that the 3-D failure mode of an embankment tends to be as wide as possible. However, the real embankments commonly fail over a finite width only. This seems to conflict with the above deterministic model. In fact, Duncan and Wright (2005) concluded that "many 3-D failures occur because soil properties and pore-water pressures vary along the length of the slope (i.e., subsurface conditions and soil properties are not uniform)" and "3-D slope failures may also occur due to the 3-D geometry of the slope". The effect of the spatial variation of soil properties on 3-D slope failure is investigated below, by using a first-order reliability method implemented in a ubiquitous spreadsheet platform.

3. First-order reliability method (FORM)

In classical FORM, the original correlated basic random variables **x** (in x-space) that define the limit state surface (LSS) or performance function g(**x**) are transformed into uncorrelated standardized normal variables **u** (in u-space), and the Hasofer and Lind (1974) index β is defined as:

$$\beta = \sqrt{\mathbf{u}{*}^T \mathbf{u}{*}} \tag{3}$$

where $\mathbf{u}^*$ is the most probable failure point or design point, denoting the point on the limit state surface $g(\mathbf{u}) = 0$ closest to the origin of u-space. Usually the design point $\mathbf{u}^*$ is not known beforehand, and the search for $\mathbf{u}^*$ can be carried out using an iterative algorithm (Rackwitz, 1976). The procedure is well explained in Ang and Tang (1984), Haldar and Mahadevan (2000), among others.

An alternative formulation of β in the x-space or in the space of standardized correlated normal variables (n-space) is presented in Low and Tang (2004, 2007):

$$\beta = \sqrt{\mathbf{n}^{*T}\mathbf{R}^{-1}\mathbf{n}^*} = \sqrt{\left[\frac{x_i{}^* - \mu_i{}^N}{\sigma_i{}^N}\right]^T \mathbf{R}^{-1} \left[\frac{x_i{}^* - \mu_i{}^N}{\sigma_i{}^N}\right]} \tag{4}$$

where, x_i^* is the design point value of the ith variable evaluated in x-space, μ_i^N and σ_i^N are equivalent normal mean and standard deviation of the ith variable, respectively, **R** is the correlation matrix, and $\mathbf{n}^*$ is the design point evaluated in n-space. μ_i^N and σ_i^N can be calculated by the Rackwitz and Fiessler (1978) transformation.

It is noted that in this paper the β and design point are obtained by a constrained search procedure, using the Excel Solver which currently contains three solving algorithms: GRG nonlinear method, Simplex method and Evolutionary method. The GRG nonlinear method is the default algorithm for the Solver which is most suitable for smooth objective functions, and it is also the preferred choice for FORM solution. One deficiency of the use of Solver for FORM is that the solution could be trapped into local minimums. In this regard, one can avoid the pitfalls by running Solver with different starting points with the auto-scaling option.

4. Modeling longitudinal spatial variation using spatial-averaging approach

The undrained shear strength c_u is commonly determined through field testing methods, and its value varies significantly over space. As a result, it is best to treat this soil property as a random field. For the study of longitudinal failure mode of long embankments, it is of particular interest to investigate the effect of longitudinal spatial variation of c_u. In this study, the exponential autocorrelation model with a longitudinal autocorrelation distance δ_z is adopted to model the random field. The point-wise c_u value has a typical coefficient of variation of 0.3. Other soil properties such as unit weight are treated as deterministic parameters.

Vanmarcke (1977b, 1980) first investigated the influence of spatial variability of soils on the 3-D slope failure modes using spatial-averaging approach. The spatial-averaging approach is based on variance reduction of random field over a local interval or area as proposed by Vanmarcke (1977a). For the example embankment model, the uncertainty of c_u on the cylindrical failure surface and the uncertainty of c_u at both end sections are considered. For a given interval length T over the longitudinal direction, the standard deviation of c_u on the cylindrical failure surface considering longitudinal spatial variation is