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# Analysis of stability of three-dimensional slopes using the rigorous limit equilibrium method

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ABSTRACT

Previously, quasi-rigorous limit equilibrium methods were applied to analyze the stability of three-dimensional slopes or landslides, which only satisfy three direction force equilibrium, and one or two direction moment equilibrium. In this paper, the rigorous limit equilibrium column method, in which inter-column forces are taken into account, is established based on six equilibrium conditions which include three direction force equilibrium conditions along coordinate axes and three direction moment equilibrium conditions around three coordinate axes. The relationship between the width of sliding body and factor of safety is determined using trust-region-reflective iterative algorithm. The value of the factor of safety is obtained using Levenberg–Marquardt least square method. Moreover, the present method can be applied to automatically search slip surface of three-dimensional landslides and to determine the factor of safety of three-dimensional landslides with the known arbitrary slip surface. Three examples are discussed to verify the robustness and precision of the present method in detail. Comparing with quasi-rigorous limit equilibrium methods which only considered four or five equilibrium conditions, the present method is more accurate and rigorous.



## 1. Introduction

Landslides are defined as the movement of a rock mass, debris or earth down a slope (Cruden, 1991). They can be triggered by a variety of external stimulus, such as earthquake shaking, intense rainfall, water level change, artificial disturbance or rapid stream erosion, and so on. Landslides are one of the major natural hazards. As shown in Fig. 1, Jipazi landslide which is mainly caused by intense rainfall occurred in July of 1982 in the province of Sichuan. The volume of the Jipazi landslide was estimated to be 15,000,000 $m^3$ and 1700 buildings had been destroyed. Jipazi landslide caused enormous losses in terms of both direct and indirect aspects.

At present, the conventional limit equilibrium method is still playing a major role in practical slope or landslide engineering. In particular, limit equilibrium methods are widely and maturely applied to analyze the stability of two-dimensional slopes or landslides. For two-dimensional slopes or landslides, factor of safety is accurate enough using limit equilibrium method which satisfies all equilibrium conditions (Duncan, 1996). However, for practical slope or landslide engineering, slopes or landslides are three-dimensional problems which are not suitable for being simplified to two-dimensional ones. Previously, when limit equilibrium columns methods were applied to study the stability of three-dimensional slopes or landslides, only partial equilibrium conditions were satisfied, which were not in the framework of rigorous limit equilibrium method. For example, only three equilibrium conditions are satisfied, which is only suitable for symmetric slopes or landslides(Hungr, 1987; Zhang, 1988; Hungr et al., 1989; Lam and Fredlund, 1993), only four or five equilibrium conditions are satisfied (Chen et al., 2001a, 2001b; Huang et al., 2002; Zhang et al., 2005). Moreover, the factor of safety, which is obtained from quasi-rigorous limit equilibrium methods, was not accurate enough to meet the engineering requirements (Baligh and Azzouz, 1975; Hovland, 1979; Hungr, 1987; Zhang, 1988; Hungr et al., 1989; Lam and Fredlund, 1993; Feng et al., 1999; Huang and Tsai, 2000; Chen et al., 2001a, 2001b; Cheng et al., 2002; Huang et al., 2002).

When rigorous limit equilibrium method is applied to analyze the stability of three-dimensional slopes or landslides, six equilibrium conditions should be satisfied, that is three direction force equilibrium conditions and moment equilibrium conditions around three axes should be satisfied. Up to now, fruitful results satisfying all of the six equilibrium conditions are few. For example, rigorous limit equilibrium method for three-dimensional slopes or landslides was investigated based on a non-column method (Zheng, 2007). The difference between rigorous and quasi-rigorous limit equilibrium stability analysis for three-dimensional slopes or landslides is compared using the method, in which normal stresses on slip surface are modified by a function with five parameters (Zhu and Qian, 2007). However, inter-column forces were not taken into account in their

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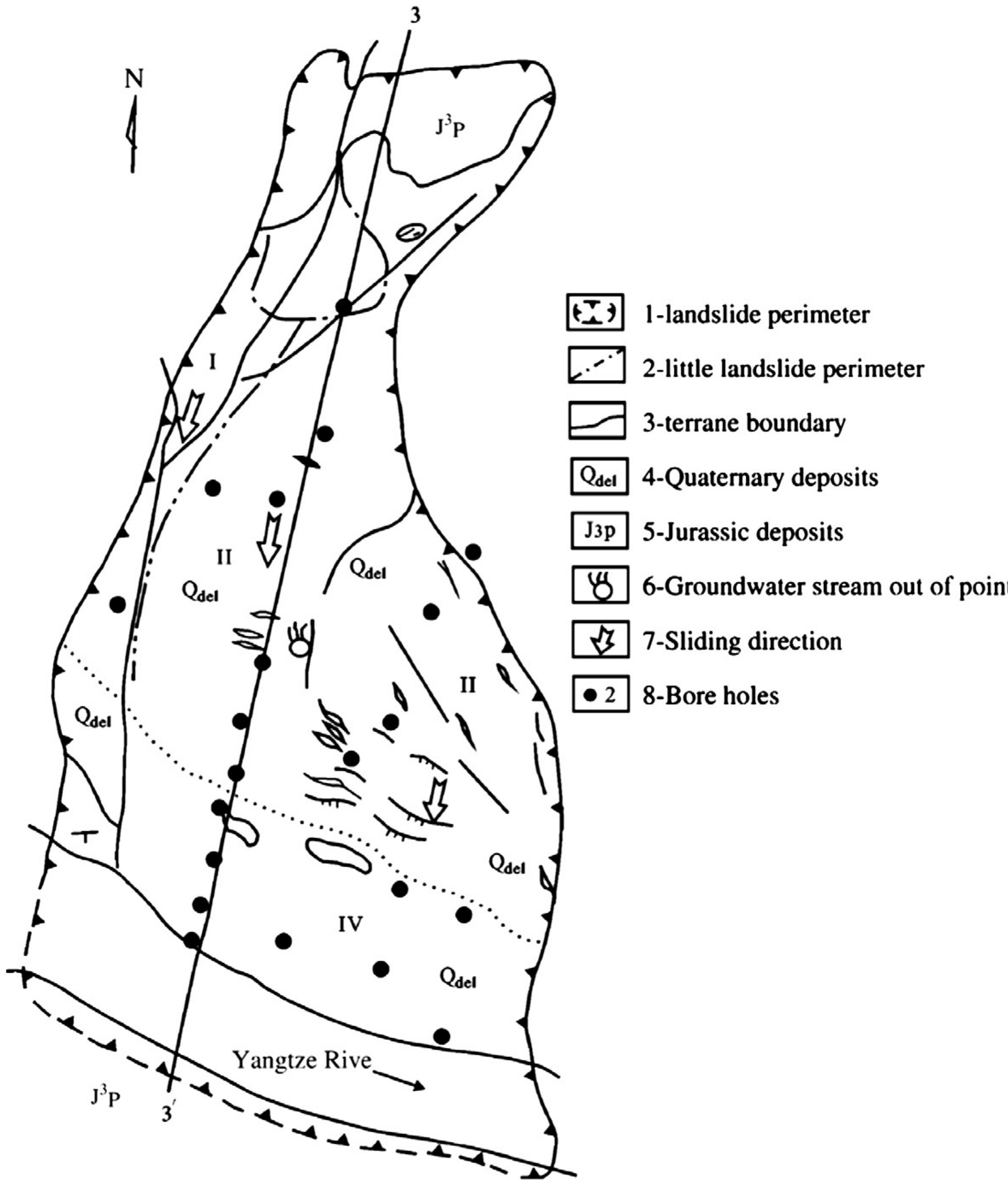


**Fig. 1.** Plane map of the Jipazi landslide.

works (Zheng, 2007; Zhu and Qian, 2007). Actually, inter-column forces significantly affect the factor of safety. In this paper, rigorous limit equilibrium method for three-dimensional slopes or landslides is proposed based on the principle of nonnegative normal force over slip surface, in which inter-column forces are considered. The relationship between the factor of safety and widths of sliding body can be determined using Levenberg–Marquardt least squares method. The factor of safety of three-dimensional slopes or landslides can be obtained using trust-region-reflective iterative algorithm.

Characteristics of the present method are summarized as follows:

(a) Six equilibrium conditions are strictly satisfied. Sum of forces along three axes are zero, and all the moments of the entire sliding body around three axes are less than $1 \times 10^{-6}$ N · m. The present method can be referred to as rigorous limit equilibrium method.

(b) The assumptions of the present method, in which inclinations of inter-column forces are not constant, are different from the Spencer method (Spencer, 1967) in which inclinations of inter-column forces are constant. In this paper, it is assumed that inclinations of inter-column forces in row direction and column direction are equal to $\pm\alpha$ and $\pm\beta$, respectively. The value of $\alpha$ and $\beta$ can strictly be obtained by numerical calculation. The values of $\alpha$ and $\beta$ represent comprehensive and average effects of the sliding body.

(c) Numerical characteristics. According to Duncan's (1996) works, non-convergence points exist in almost all examples of rigorous limit equilibrium methods. In this paper, the non-linear relationship between the factor of safety and the width of sliding body is obtained using Levenberg–Marquardt least square method. Factor of safety in arbitrary intervals can be obtained by the