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Probabilistic characterization of Young's modulus of soil using equivalent samples

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ABSTRACT

Several probability-based design codes (e.g., load and resistance factor design (LRFD) codes and Eurocode 7) have been developed and implemented around the world recently. A characteristic (or nominal) value of soil/rock properties is used in these design codes, and it is typically defined as a pre-specified quantile (e.g., mean or lower 5% quantile) of the statistical distribution of the soil properties. This poses a challenge in the implementation of the design codes, because the number of soil/rock property data obtained during site investigation is generally too sparse to generate meaningful statistics, rendering proper selection of the characteristic value a very difficult task. This paper aims to address this challenge by developing a Markov Chain Monte Carlo Simulation (MCMCS)-based approach for probabilistic characterization of undrained Young's modulus, E_u, of clay using standard penetration tests (SPT). Prior knowledge (e.g., previous engineering experience) and project-specific test data (e.g., SPT test data) are integrated probabilistically under a Bayesian framework and transformed into a large number, as many as needed, of equivalent samples of E_u. Subsequently, conventional statistical analysis is carried out to estimate statistics of E_u, and the characteristic value of the soil property is selected accordingly. Equations are derived for the proposed approach, and it is illustrated and validated using real SPT and pressuremeter test data at the clay site of the US National Geotechnical Experimentation Sites (NGES) at Texas A&M University.

1. Introduction

During the past two decades, several probability-based design codes have been developed and implemented around the world, such as the load and resistance factor design (LRFD) or multiple resistance factor design (MRFD) for foundations (Barker et al., 1991; Phoon et al., 1995, 2003a, 2003b; Paikowsky et al., 2004, 2010) in the United States, the National Building Code for foundations (Becker, 1996) in Canada, the Eurocode 7 (BSI, 2010) in Europe, and the Geocode 21 (i.e., JGS4001 (Japanese Geotechnical Society, 2006; Honjo et al., 2010)) in Japan. Probability theory is nominally used in the development of these design codes to achieve target degrees of reliability through proper consideration of geotechnical-related uncertainties that arise in loads, geological site interpretations, geotechnical properties, computational models, etc. Consider, for example, the uncertainty in soil/rock properties. A characteristic (or nominal) value of soil/rock properties is used in the design codes, and it is typically defined as a pre-specified quantile (e.g., mean or lower 5% quantile) of the statistical distribution of the soil/rock properties. This poses a challenge in the implementation of the design codes because of the difficulty in selecting the appropriate characteristic value in geotechnical practice. Note that the number of soil/rock property data obtained during geotechnical site investigation is generally too sparse to generate meaningful statistics (e.g., mean, standard deviation, and other high order moments), rendering proper selection of the characteristic value a very difficult task (e.g., Cao, 2012; Luo et al., 2013).

This paper aims to address this challenge by developing a Markov Chain Monte Carlo Simulation (MCMCS)-based approach that generates a large number of equivalent soil/rock property data for proper statistical characterization of the soil/rock property and the subsequent selection of its characteristic value. The proposed approach takes advantages of available site information prior to the project, which is referred to as “prior knowledge” in this study, and utilizes both the prior knowledge and project-specific information from test borings, in-situ testing, and/or laboratory testing (Clayton et al., 1995; Mayne et al., 2002). Prior knowledge include, but are not limited to, maps and surveys, local experience, engineering judgment, visual observations, and published reports and studies (Wang et al., 2010a). They are generally collected during preliminary stages (e.g., desk study or site reconnaissance) of geotechnical site characterization (Clayton et al., 1995; Mayne et al., 2002). Then, both prior knowledge and project-specific information are integrated probabilistically under a Bayesian framework. The Bayesian framework provides a rational vehicle to combine systematically information from different sources (e.g., Ang and Tang, 2007; Marache et al., 2009; Wang et al., 2010a; Ching et al., 2010), and it has several successful applications in geotechnical engineering, such as characterization of model uncertainty (e.g., Zhang et al., 2004, 2009; Najjar and Gilbert, 2009; Zhang

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et al., 2012), back analysis of soil parameters using observed performance of geotechnical structures (e.g., Zhang et al., 2010; Juang et al., 2013), evaluation of the reliability of the soil-water characteristic curve (e.g., Chiu et al., 2012), and probabilistic characterization of effective friction angle of sand using cone penetration tests (Wang et al., 2010a; Cao et al., 2011; Cao and Wang, 2013).

This paper focuses on probabilistic characterization of the undrained Young's modulus, E_u, of clay using standard penetration tests (SPT). It starts with probabilistic modeling of the inherent variability of E_u and transformation uncertainty associated with the regression between E_u and SPT N values, followed by derivation of the probability density function (PDF) of E_u based on prior knowledge and project-specific SPT data under a Bayesian framework. Then, a large number of equivalent samples of E_u are generated from this PDF using MCMCS. Conventional statistical analysis of the equivalent examples is subsequently carried out to determine the statistics of E_u and its characteristic value. Implementation procedure of the proposed equivalent sample approach is described, and the computer code for the proposed approach is included as an appendix to assist in its applications by engineers. As an illustration, the proposed approach is applied to characterize probabilistically the E_u at the clay site of the US National Geotechnical Experimentation Sites (NGES) at Texas A&M University. In addition, sensitivity studies are performed to explore the effect of the amount of project-specific test data and prior knowledge on the probabilistic characterization of soil properties.

2. Uncertainty modeling

2.1. Inherent variability

Geotechnical materials are natural materials, and their properties are affected by various factors during their formation process, such as the properties of their parent materials, weathering and erosion processes, transportation agents, and conditions of sedimentation (Vanmarcke, 1977; Phoon and Kulhawy, 1999a; Baecher and Christian, 2003; Mitchell and Soga, 2005). The properties of geotechnical materials therefore vary spatially, and such inherent variability is independent of the state of knowledge regarding geotechnical properties and cannot be reduced as the knowledge improves (Baecher and Christian, 2003). Consider, for example, the undrained Young's modulus, E_u, within a clay layer. To model explicitly the inherent variability, E_u is represented by a lognormal random variable with a mean μ and standard deviation σ, and it is defined as (e.g., Ang and Tang, 2007; Au et al., 2010; Cao, 2012):

$$E_u = \exp(\mu_N + \sigma_N z) \tag{1}$$

in which z is a standard Gaussian random variable; $\mu_N = \ln\mu - \frac{1}{2}\sigma_N^2$ and $\sigma_N = \sqrt{\ln(1 + (\sigma/\mu)^2)}$ are the mean and standard deviation of the logarithm (i.e., $\ln(E_u)$) of E_u, respectively. $\ln(E_u)$ is normally distributed, and it is expressed as:

$$\ln(E_u) = \mu_N + \sigma_N z \tag{2}$$

Note that both the σ of E_u and the σ_N of $\ln(E_u)$ represent the inherent variability of the undrained Young's modulus within the clay layer. Because of the physical meaning of Young's modulus, E_u is a continuous variable and must be strictly non-negative. It is therefore frequently modeled as a lognormal random variable in geotechnical literature (e.g., Lumb, 1966; Fenton, 1999; Fenton and Griffiths, 2005). Although this study follows these previous studies to model E_u as a lognormal random variable, the proposed approach is general and applicable for different distribution types of random variables.

2.2. Transformation uncertainty

The undrained Young's modulus of clay can be measured directly using pressuremeter tests, which are generally considered as one of the most accurate measurements for E_u, but are certainly expensive and time consuming (Mair and Wood, 1987; Briaud, 1992; Wang and O'Rourke, 2007). More frequently and particularly for projects with medium or relatively small sizes, the E_u is usually estimated indirectly from other in-situ tests, such as SPT tests (Kulhawy and Mayne, 1990; Clayton, 1995; Mayne et al., 2002). The E_u value of the tested soil is obtained by means of regression between the E_u and the N values measured during SPT tests. Fig. 1 shows a regression model between the E_u measured by pressuremeter tests and SPT N values (Ohya et al., 1982; Kulhawy and Mayne, 1990; Phoon and Kulhawy, 1999b):

$$E_u/p_a = 19.3N^{0.63} \tag{3}$$

in which p_a is the atmospheric pressure (i.e., 0.1 MPa). Eq. (3) can be rewritten in a log–log scale as:

$$\xi = \ln(N) = a\ln(E_u) + b + \varepsilon \tag{4}$$

in which $\xi = \ln(N)$ denotes the SPT N values in a log scale; $a = 1.587$, $b = -1.044$, and ε is a Gaussian random variable with a zero mean and a standard deviation $\sigma_\varepsilon = 1.352$ (Kulhawy and Mayne, 1990; Phoon and Kulhawy, 1999b). The last term ε represents a modeling scatterness or transformation uncertainty associated with the regression equation. Note that the original regression between E_u and SPT N values, i.e., Eq. (3), was developed by Kulhawy and Mayne (1990) using the data provided by Ohya et al. (1982), and it has been inverted to obtain Eq. (4) for development of the approach proposed in this study. It is also worthwhile to note that the proposed approach is applicable to different choices of regression equations, and it provides engineers with flexibility to choose a regression equation deemed appropriate for the projects/sites in hand based on their engineering judgments. Generally speaking, regression equations developed locally for a particular geological setting tend to have relatively small variation/uncertainty, as opposed to the relatively large variation/uncertainty for those developed based on global data that may involve many different geological settings (Phoon and Kulhawy, 1999b; Zhang et al., 2004; Ching and Phoon, 2012). Engineers therefore may choose the local regression equations, if available, when using the proposed approach.

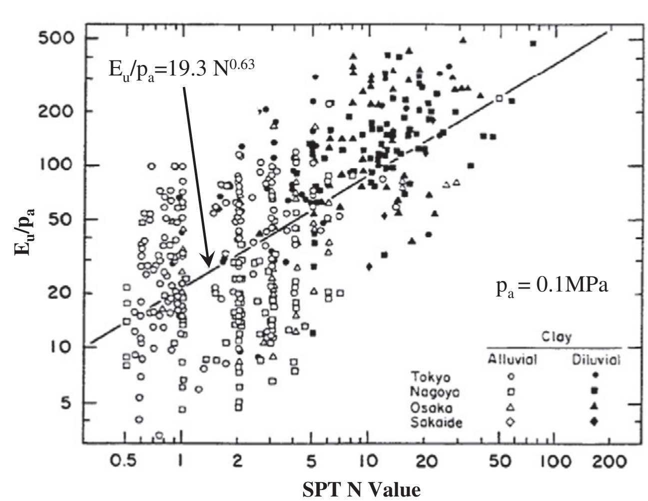

Fig. 1. Regression between SPT N value and undrained Young's modulus of clay (after Ohya et al., 1982; Kulhawy and Mayne, 1990; Phoon and Kulhawy, 1999b).