Engineering Geology 166 (2013) 140–151

Contents lists available at ScienceDirect

Engineering Geology

journal homepage: www.elsevier.com/locate/enggeo

Generalized unconfined seepage flow model using displacement based formulation

Pedro Navas [1], Susana López-Querol *

Department of Civil Engineering, University of Castilla - La Mancha, Avda. Camilo Jose Cela s/n, 13071 Ciudad Real, Spain
School of Computing and Technology, University of West London, St Mary Campus, Ealing, London

ARTICLE INFO

Article history:
Received 14 June 2013
Received in revised form 10 September 2013
Accepted 22 September 2013
Available online 29 September 2013

Keywords:
Seepage
Finite element modeling
Biot's equations
Earth dams
Drawdown
u–*w* formulation

ABSTRACT

Modeling seepage along with the mechanical response of deformable Earth dams under transient conditions is a very complicated task, since it involves coupling between different phases, computation of free surface variables in time, and thus it requires algorithms for integration in time. These aspects represent a combination of several problems, which are usually undertaken in a separate way, uncoupling mechanical response and flow through the porous media. When such computations are carried out, most of the times rigid solid skeleton is considered without a comprehensive analysis of the degree of accuracy achieved with such assumptions. Moreover, it is rather difficult to find in the literature coupled formulations under transient conditions. In this paper, a numerical finite element, coupled, transient model for analyzing unconfined seepage through Earth dams is presented. This model is based on Biot's equations, in terms of displacements (so called *u*–*w* formulation). The iterative procedure to obtain free surfaces by changing impermeability boundary conditions is implemented in this model. This generalized model is validated against several cases found in the literature. After that, several relevant aspects of the particular problem of fast emptying of a reservoir, and the calculation of the limiting drawdown speed for not compromising the Earth dam safety, are explored. Thus, the influences of different drawdown speeds, soil permeability values, stiffness and geometries in a theoretical rectangular Earth dam have been analyzed in terms of effective vertical stress changes at relevant points inside the dam. In summary, all these studied cases show the suitability of the presented methodology for evaluating such situations in real Earth dams, and give hints on the more significant aspects to be considered in the Earth dam design.

1. Introduction

During the last forty years, plenty of methods for determining free surface in unconfined seepage problems through porous media have been developed, based on numerical schemes. In most of these approaches, rigid solid skeleton and steady condition hypotheses are usually assumed for undertaking this computation, neglecting the influence of these aspects in the final results. In addition, the hypothesis of totally dry or saturated state is also a usual assumption.

The first numerical methodologies for undertaking such problems consisted of generating a tentative mesh in the spatial domain, solving the seepage governing equation (rigid media, steady conditions), verifying the free surface conditions (which at the same time is boundary and flow line), and adapting the existing mesh in an iterative manner until reaching a final consistent solution, with a clear separation of dry and saturated spatial domains (Finn, 1967; Taylor and Brown, 1967; Neuman and Witherspoon, 1970). These procedures are very much time consuming, and were modified in the way of slightly moving the nodes close to the tentative free surface location, until convergence, and hence, escaping from the necessity to generate a new mesh in each iteration, but just changing a little bit the existing one (Oden and Kikuchi, 1980). These adaptative mesh methods often lead to inaccurate calculations, since the elements close to the boundary can be extremely distorted at the end of the computation process. Aiming to improve this aspect, a new approach, consisting of keeping constant the spatial domain, but making variable the soil permeability above and below the free surface, was adopted (Desai, 1976; Bathe and Khoshgoftaar, 1979; Bardet and Tobita, 2002; Kazemzadeh-Parsi and Daneshmand, 2012).

The first methodologies in which the solid skeleton was considered as deformable (elastic) were those developed by Lacy and Prevost (1987) and Borja and Kishnani (1991), with variable permeability respectively below and above the free surface. Such procedures meant a major computational effort, since, in each iteration, the stiffness matrix must again be obtained, and inverted.

All the above mentioned models were developed for steady flow conditions. Methods for variable flow have been also proposed by Herbert (1968), and, more recently, by Herreros et al. (2006), who applied a completely different approach to determine the free surface, namely

* Corresponding author. Tel.: +34 926295300x6263.
E-mail addresses: penavasal@hotmail.com (P. Navas), mariasusana.lopez@uclm.es (S. López-Querol).
[1] Tel.: +34 926295300x6237.

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http://dx.doi.org/10.1016/j.enggeo.2013.09.009

the level set technique, which has proved to be a very efficient tool to solve transient flow problems.

All the above mentioned methodologies are formulated in terms of water heads, focusing more in the fluid behavior, and almost neglecting the coupling with the solid skeleton in most of the cases. The novelty of the new methodology presented in this paper is, on the one hand, a coupled formulation of the problem of seepage through porous media, based on displacements of both solid and fluid phases, the so called u–w formulation. On the other hand, a recently developed methodology to obtain free surfaces, iteratively changing the impermeability boundary conditions, and using constant permeability (i.e., constant spatial domain without needing to iterate the stiffness matrix), has been implemented, making the model very efficient (López-Querol et al., 2011).

The paper begins showing the governing equations along with the time integration scheme followed to compute the evolution in time of the solution, under transient conditions. Validations of the method, against previously published results, considering deformable solid skeleton and transient conditions, are also included.

Finally, the influence of several factors which might compromise an Earth dam safety in a fast emptying of the reservoir is explored, trying to obtain practical conclusions and relevant hints in the Earth dam's design.

2. Model features

2.1. Governing equations

The present problem has been undertaken using the Biot's equations (Biot, 1956; Zienkiewicz et al., 1999; López-Querol et al., 2008). These equations are based on formulating the mechanical behavior of a solid–fluid mixture, the coupling between different phases, and the continuity of flux through a differential domain. Thus, Eqs. (1) and (2) respectively account for the momentum equilibrium at the soil–fluid mixture, and only the fluid phase. Eq. (3) represents the continuity equation:

$$S^T \cdot d\sigma - \rho \cdot d\ddot{u} - \rho_f \cdot d\ddot{w} + \rho \cdot db = 0 \tag{1}$$

$$-\nabla dp_w - K^{-1} \cdot d\dot{w} - \rho_f \cdot d\ddot{u} - \frac{\rho_f}{n} \cdot d\ddot{w} + \rho_f \cdot db = 0 \tag{2}$$

$$\nabla^T d\dot{w} + m^T \cdot d\dot{\varepsilon} + \frac{d\dot{p}_w}{Q} = 0. \tag{3}$$

In the above equations, u denotes displacement of the solid skeleton, and w represents the relative displacement of the fluid phase with respect to the solid one, which is determined by Eq. (4):

$$w = n \cdot (U - u). \tag{4}$$

In these expressions, U is the absolute displacement of the fluid phase; n is the porosity of the soil, ρ and ρ_f respectively denote mixture and fluid phase densities, p_w represents pore water pressure, tb is the external acceleration vector and K means permeability matrix, expressed in units $[m^3] \cdot [s]/[kg]$. Moreover, Q is the volumetric compressibility of the mixture, and S represents a matrix operator, which, in 2D problems, is defined as Eq. (5):

$$S = \begin{pmatrix} \frac{\partial}{\partial x} & 0 \\ 0 & \frac{\partial}{\partial y} \\ \frac{\partial}{\partial y} & \frac{\partial}{\partial x} \end{pmatrix} \tag{5}$$

S also relates incremental strain vector, $d\varepsilon$ and incremental solid phase displacement vector, du:

$$d\varepsilon = S \cdot du. \tag{6}$$

In Eq. (3), m represents the identity vector, which in 2D is expressed as:

$$m = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}. \tag{7}$$

The adopted sign criterion is: extension is positive for both stress and strains, σ and ε, and conversely, pore water pressure p_w is considered positive in compression.

By means of Terzaghi's effective stress decomposition rule:

$$\sigma = \sigma' - m \cdot p_w \tag{8}$$

where σ' and σ respectively indicate effective and total stress vectorial expression of the corresponding tensors (Zienkiewicz et al., 1999).

If linear elasticity is adopted as constitutive law, the relationship between stresses and strains, expressed in its incremental form, is governed by:

$$d\sigma' = D^e \cdot d\varepsilon \tag{9}$$

where D^e denotes the elastic tensor, which, for plane strain conditions, is given by:

$$D^e = \frac{\lambda}{\nu} \cdot \begin{pmatrix} 1-\nu & \nu & 0 \\ \nu & 1-\nu & 0 \\ 0 & 0 & \frac{1-2\cdot\nu}{2} \end{pmatrix} \tag{10}$$

where ν indicates the Poisson's ratio, λ is the Lame's constant:

$$\lambda = \frac{2 \cdot G \cdot \nu}{1 - 2 \cdot \nu} \tag{11}$$

and G is the elastic shear modulus.

Rearranging the above equations, Eq. (1) yields:

$$S^T \cdot D^e \cdot S \cdot du - \nabla^T dp_w - \rho \cdot d\ddot{u} - \rho_f \cdot d\ddot{w} + \rho \cdot db = 0. \tag{12}$$

2.2. u–w formulation

The problems dealt with in this research are considered 2D under plane strain conditions. Hence, at each node, the number of degrees of freedom is 5: vectors u and w (with two components each) and the scalar p_w. Trying to improve the numerical efficiency of this methodology, it is of paramount importance to reduce this number of unknowns. To do that, the most extended trend is the u–p_w formulation, which departs from the assumption of neglecting $d\ddot{w}$. By so doing, the number of degrees of freedom drastically decreases, particularly in 3D problems.

Another possibility is the so called u–w formulation, also known as "complete" formulation, since it does not require additional assumptions. Eq. (3) is integrated in time, and dp_w is then substituted into Eqs. (1) and (2), yielding:

$$S^T \cdot D^e \cdot S \cdot du + Q \cdot \nabla\left(\nabla^T du\right) + Q \cdot \nabla\left(\nabla^T dw\right) - \rho \cdot d\ddot{u} - \rho_f \cdot d\ddot{w} + \rho \cdot db = 0 \tag{13}$$

$$Q \cdot \nabla\left(\nabla^T du\right) + Q \cdot \nabla\left(\nabla^T dw\right) - K^{-1} \cdot d\dot{w} - \rho_f \cdot d\ddot{u} - \frac{\rho_f}{n} \cdot d\ddot{w} + \rho_f \cdot db = 0. \tag{14}$$

The above set of equations is solved in the spatial domain using a finite element scheme, applying Galerkin's method (Ottosen and Petersson, 1992; Zienkiewicz and Taylor, 2000). Triangular finite elements, using quadratic approximation (6 nodes each element), are