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First-order reliability analysis of slope considering multiple failure modes

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ABSTRACT

This work studies the reliability analysis of a slope that considers multiple failure modes. The analysis consists of two parts. First, significant failure modes that contribute most to system reliability are determined. The so-called barrier method proposed by Der Kiureghian and Dakessian to identify significant failure modes successively is employed. Second, the failure probability for the slope is estimated on the basis of identified significant failure modes and corresponding design points. For reliability problems entailing multiple design points, failure probability can be estimated by the multi-point first-order reliability method (FORM), which gives the probability of the union of approximate events. FORM approximations at each design point and a subsequent series system reliability analysis are employed to estimate failure probability. Application of the procedure is illustrated through example problems. The results show that the applied procedure is able to efficiently consider various failure modes caused by stratifications and variations in soil properties in probabilistic slope stability assessments.

1. Introduction

In slope reliability analysis, numerous slip surfaces may be present. If failure along any individual slip surface is viewed as a failure mode, the problem of slope stability can be considered as a series system with infinite failure modes in the sense that failure of the slope occurs if just one of the potential slip surfaces fails. However, it is not possible to determine system reliability accurately for slope stability problems. The probability of failure for the most critical slip surface is therefore commonly used to estimate system failure probability. This approach assumes that the probabilities of failure along differing slip surfaces are highly correlated (Chowdhury and Xu, 1995). In the case of highly correlated modes, the contribution to system failure probability from failure surfaces other than that associated with the maximum failure probability may be small, even though the modes are infinitely many (Cornell, 1967). However, as noted by Cornell (1971), the overall failure probability of a slope may be greater than that along any individual slip surface when the correlation between differing potential slip surfaces is not strong. Some researchers have therefore considered the system reliability of slopes by studying several slip surfaces.

Ditlevsen's (1979) bounds method is widely used to calculate the system failure probability of slopes based on limit equilibrium methods (Oka and Wu, 1990; Chowdhury and Xu, 1995; Low et al., 2011; Ji and Low, 2012). Griffiths and Fenton (2004) and Huang et al. (2010) have used Monte Carlo simulation (MCS) to calculate the system failure probability of a slope based on finite element models.

Hong and Roh (2008) and Cho (2010) have dealt with the system aspect of the slope in reliability analysis by defining a limit state for the system as a function of the minimum factor of safety for all potential slip surfaces.

Ching et al. (2009) have suggested a method based on the importance sampling (IS) technique to efficiently estimate the system failure probability of slope stability for circular slip surfaces based on the ordinary method of slices. They analyzed several slopes, and the results were compared with the single-mode first-order reliability method (FORM). From the results of the numerical examples, they concluded that the methods based on FORM may underestimate the failure probability depending on the number of failure modes.

The present paper describes a study of the failure probability of a slope that considers multiple failure modes to obtain further insight into this probability. The reliability analysis consists of two parts. In the first part, significant failure modes that make the greatest contribution to system reliability are determined. The so-called barrier method proposed by Der Kiureghian and Dakessian (1998) to successively identify significant failure modes and corresponding design points is employed. Reliability analysis is then carried out to estimate the failure probability for the slope on the basis of the identified significant failure modes. For reliability problems entailing multiple design points, failure probability can be estimated by the multi-point FORM, which gives the probability of the union of approximate events. FORM approximations at each design point and subsequent series system reliability analysis are employed to estimate the failure

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probability. Application of the procedure is illustrated through example problems.

2. Deterministic slope stability analysis

2.1. Limit equilibrium method

Slope stability problems are commonly analyzed using limit equilibrium methods of slices. The failing soil mass is divided into a number of vertical slices to calculate the factor of safety, which is defined as the ratio of the resisting shear strength to the mobilized shear stress needed to maintain static equilibrium. The static equilibrium of the slices and the mass as a whole are used to solve the problem. However, all methods of slices are statically indeterminate and, as a result, require assumptions in order to solve the problem. The approach presented here adopts Bishop's simplified method, which is widely accepted as being reasonably accurate and is applicable to the failure surface of a circular shape.

2.2. Search for critical failure surface

Limit equilibrium methods require that the critical failure surface be determined as part of the analysis. The problem of locating the critical circular surface can be formulated as an optimization problem:

$$\min_{surface} F_s(x_c, y_c, R) \tag{1}$$

where F_s is the objective function (the factor of safety), x_c and y_c are the coordinates of the center of the circle, and R is the radius of the circle.

Although a circular trial slip surface can be described as a function of three shape variables, it could be defined as a series of straight segments. The location of the segment vertices is determined by shape variables and by the location of a hard stratum that the slip surface cannot penetrate. Treatment of a circular slip surface is straightforward because there are only three location parameters. Eq. (1) is a type of unconstrained optimization problem that has no equality or inequality conditions. The Broyden–Fletcher–Goldfarb–Shanno method, widely acknowledged to be efficient, was applied in this study in order to search for the critical circular slip surface. Kim and Lee (1997) have described this procedure in detail.

3. Probabilistic slope stability analysis

3.1. Limit state function

The problem of probabilistic slope stability analysis is formulated by a vector, $\mathbf{x}=[x_1,x_2,x_3,\ldots,x_n]$, that represents a set of random variables. From the uncertain variables, a limit state function $g(\mathbf{x})$ is formulated to describe the limit state in the space of $\mathbf{x}$. In n-dimensional hyperspace of the basic variables, $g(\mathbf{x})=0$ is the boundary between the region in which the target factor of safety is not exceeded and the region in which it is exceeded.

The limit state function for the slope stability is usually defined as

$$g(\mathbf{x}) = F_s - 1.0. \tag{2}$$

Bishop's simplified method is used to describe the limit state function of Eq. (2) by calculating F_s for the failure surface.

The probability of failure of the slope is then given by the following integral (Baecher and Christian, 2003):

$$P_f = P[g(\mathbf{x} \le 0)] = \int_{g(\mathbf{x})\le 0} f_{\mathbf{x}}(\mathbf{x})\mathbf{dx} \tag{3}$$

where $f_{\mathbf{x}}(\mathbf{x})$ denotes the joint probability density function, and the integral is over the failure domain.

For slope stability problems, direct evaluation of the n-fold integral is virtually impossible. Therefore, approximate techniques have been developed to evaluate this integral.

3.2. First-order reliability method

First-order reliability evaluation of Eq. (3) is accomplished by transforming the uncertain variables, $\mathbf{x}$, into uncorrelated standard normal variables, $\mathbf{u}$. The primary contributor to the probability integral in Eq. (3) is the part of the failure region ($G(\mathbf{u})\le 0$, for which $G(\mathbf{u})$ is the limit state function in the transformed normal space) closest to the origin. The design point is defined as point $\mathbf{u}^*$ in the standard normal space located on the limit state function ($G(\mathbf{u})=0$) with the maximum probability density attached to it. Therefore, the design point, which is the point closest to the origin in the failure region, is an optimum point at which to approximate the limit state surface. The probability approximated at the design point is

$$P[g(\mathbf{x})\le \mathbf{0}] \approx \Phi(-\beta) \tag{4}$$

where β is the reliability index defined by the distance from the origin to the design point and Φ is the standard normal cumulative density function.

In FORM, a tangent hyperplane is fitted to the limit state surface at the design point. Therefore, the most important and challenging step in the method is finding this point. The design point is the solution of the following nonlinear constrained optimization problem:

$$\min||\mathbf{u}|| \text{subject to } G(\mathbf{u}) = 0. \tag{5}$$

Generally, the FORM approximation gives a reasonable result for a limit state function with only one global design point. However, this is not the case when there are other local design points on the limit state surface. In this case, multiple design points make important contributions to the total system probability of failure, and significant errors will be induced if one of these points is missing. Unfortunately, conventional gradient-based optimization algorithms used in connection with FORM are able to identify only one design point and provide no information on other potential design points.

4. Identification of significant failure modes

Finding the entire set of relevant design points is, in general, a challenging problem for classical gradient-based optimization methods.

If multiple design points exist, or if there are contributions from other regions around local minimums besides the region around a single design point, these methods may fail to provide a correct estimation of failure probability. To handle problems with multiple design points, two steps need to be taken: (1) search techniques to find all design points and (2) system reliability analysis that takes into account the correlation of the piecewise approximation of the limit state surface based on these design points (Wei, 2006).

The arbitrary failure set defined in Eq. (3) may present large calculation difficulties. As suggested by Ditlevsen and Madsen (1996), this set can be approximated by multiple first-order approximations, as shown in Fig. 1. A reliability calculation can then be performed with less difficulty for a simpler failure set. To use this method, all local minima need to be determined in advance, and the quality of the solution depends on the accuracy of these approximations (Wei, 2006).

A simple approach to determining significant failure modes is simulation, where realizations of relevant random variables are simulated. Corresponding to each set of realizations, failure modes are determined by slope stability analysis. The process is repeated until sufficient failure modes are discovered. Although this approach is straightforward, simulations require large computational efforts. In addition, it is difficult to guarantee that all the significant modes have been found. Zhang et al.