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Exact solutions of some nonlinear partial differential equations using the variational iteration method linked with Laplace transforms and the Padé technique

Tamer A. Abassy[a], Magdy A. El-Tawil[b,*], H. El-Zoheiry[b]

[a] *Department of Basic Science, Benha Higher Institute of Technology, Benha University, 13512, Egypt*
[b] *Department of Engineering Mathematics and Physics, Faculty of Engineering, Cairo University, Giza, Egypt*

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Abstract

In this paper, the variational iteration method (VIM) is reintroduced with Laplace transforms and the Padé technique treatment to obtain closed form solutions of nonlinear equations. Some examples, including the coupled Burger's equation, compacton $k(n,n)$ equation, Zakharov–Kuznetsov $\mathrm{Zk}(n,n)$ equation, and KdV and mKdV equations are given to show the effectiveness of the coupled VIM–Laplace–Padé and VIM–Padé techniques.

Keywords: Nonlinear partial differential equations; Variational iteration method; Compactons; Laplace transform; Padé approximant

1. Introduction

In the last few decades, active research efforts were focused on nonlinear dynamical systems that emerge in various fields, such as fluid mechanics, plasma physics, biology, hydrodynamics, solid-state physics and optical fibers. These nonlinear phenomena are often related to nonlinear wave equations. In order to better understand these phenomena as well as further apply them in practical scientific research, it is important to seek their exact solutions.

Many powerful methods have been developed for this purpose, such as the Backlund transformation [1,2], Hirota's bilinear method [3], Darboux transformation [4], symmetry method [5], the inverse scattering transformation [6], the tanh method [7–9], the sine–cosine method [10,11], the Adomian decomposition method [12] and other asymptotic methods for strongly nonlinear equations [13].

The variational iteration method (VIM) was proposed by He [14,15]. It was successfully applied to autonomous ordinary differential equations in [16], to nonlinear polycrystalline solids in [17], to the construction of solitary solutions and compacton-like solutions for nonlinear dispersive equations [18], and in other fields.

* Corresponding author. Tel.: +20 2 5625129; fax: +20 2 5723486.
E-mail addresses: tamerabassy@yahoo.com (T.A. Abassy), magdyeltawil@yahoo.com (M.A. El-Tawil), hanafiel_zoheiry@hotmail.com (H. El-Zoheiry).

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VIM has been modified by the authors in [19]. There are many different approaches to the construction of the initial guess. In the case $u_0(x) = u(x, 0)$, the modified VIM [19] leads to better solutions in less time for a class of nonlinear problems, while preventing the repeated and unneeded terms required while introducing modified iterative equations. In [20,21], the Adomian decomposition method was linked with the Padé technique to increase the domain of convergence of the truncated series solution of Adomian alone. The authors have improved VIM results by linking it with the Padé technique in [22], which increases the domain of convergence of the truncated series solutions for many important nonlinear equations.

In this paper, the authors extend the work done in [20] and extract the exact solution from VIM results using Padé approximants and Laplace transforms for some nonlinear problems, namely compactons $K(n, n)$, Zk(n, n), coupled Burger's system, and KdV and mKdV equations.

2. Variational iteration method

To show the basic concepts of VIM, consider the following general non-linear partial differential equation

$$\begin{aligned} &Lu(x,t) + Ru(x,t) + Nu(x,t) = 0, \\ &u(x,0) = f(x), \end{aligned} \tag{1}$$

where, $L = \frac{\partial}{\partial t}$, R is a linear operator and $Nu(x, t)$ is the nonlinear term. $Ru(x, t)$ and $Nu(x, t)$ don't have partial derivatives with respect to t.

According to the variational iteration method [13,14], an iteration formulation can be constructed in the following way

$$U_{n+1}(x,t) = U_n(x,t) + \int_0^t \lambda \left\{ LU_n + \overbrace{RU_n}^{\sim} + \overbrace{NU_n}^{\sim} \right\} \mathrm{d}\tau \tag{2}$$

where λ is a general Lagrange multiplier, which can be identified optimally via variational theory, $\overbrace{RU_n}^{\sim}$, and $\overbrace{NU_n}^{\sim}$ are considered as restricted variations, i.e. $\delta \overbrace{RU_n}^{\sim} = 0$, $\delta \overbrace{NU_n}^{\sim} = 0$, and its stationary conditions can be obtained as:

$$\begin{aligned} &1 + \lambda|_{\tau=t} = 0, \\ &\lambda' = 0. \end{aligned} \tag{3}$$

The Lagrange multiplier, therefore, can be identified as $\lambda = -1$, and the following variational iteration formula can be obtained as

$$U_{n+1} = U_n - \int_0^t \{L\,(U_n) + R(U_n) + NU_n\}\,\mathrm{d}\tau. \tag{4}$$

The second term on the right is called the correction term. Eq. (4) can be solved iteratively using $U_0(x)$ as the initial approximation, with possible unknowns.

3. Modified variational iteration method

In [19], the modified variational iteration method (MVIM) was introduced, which approximately solves equations in the form of Eq. (1).

Following the same procedure as VIM's for calculating the Lagrange multiplier [13,14], we obtain the following iteration formula [19]:

$$U_{n+1} = U_n - \int_0^t \{R(U_n - U_{n-1}) + (G_n - G_{n-1})\}\,\mathrm{d}\tau, \tag{5}$$

where $U_{-1} = 0$, $U_0 = f(x)$ and $G_n(x, t)$ is calculated from the relation

$$NU_n(x,t) = G_n(x,t) + O(t^{n+1}), \tag{6}$$