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Periodic solutions for some strongly nonlinear oscillations by He's variational iteration method

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Abstract

In this paper, we implement a new analytical technique, He's variational iteration method, for some strongly nonlinear oscillations. A correction functional is constructed by using a general Lagrange multiplier, which can be identified via the variational theory. The obtained approximate solutions and periods are compared with exact or numerical results to verify the effectiveness and accuracy of the method.

Keywords: Nonlinear oscillations; Variational iteration method; Periodic solution

1. Introduction

It is well known that the perturbation method is one of the commonly used quantitative methods for analysing nonlinear problems [1,2]. The perturbation method is valid, in principle, only for problems containing small parameters. Its basic idea is to transform, by means of small parameters, a nonlinear problem into an infinite number of linear subproblems, or a complicated linear problem into an infinite number of simpler ones. Therefore, the small parameter plays a very important role in the perturbation method. It determines not only the accuracy of the perturbation approximations, but also the validity of the perturbation method itself. However, in science and engineering, there exist many nonlinear problems which do not contain any small parameters, especially those with strong nonlinearity. Thus it is necessary to develop and improve some nonlinear analytical techniques which are independent of small parameters. There exists a wide body of literature dealing with the problem of approximate solutions to nonlinear equations with various different methodologies. Many different approaches have been proposed, such as the perturbation iteration method [3,4], the modified Lindstedt–Poincare method [5], the Adomian's decomposition method [6], etc. The variational iteration method (VIM) was proposed by Ji-Huan He [7,8], and has been proved by many authors to be a powerful mathematical tool for various kinds of nonlinear problems. A variational principle for nano thin film lubrication was established by the semi-inverse method by He [9]. The variational iteration method is used for solving three types of nonlinear partial differential equations, such as the coupled Schrodinger-KdV, generalized

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KdV and shallow water equations by Abdou and Soliman [10], for solving generalized Burger–Fisher and Burger equations by Moghimi and Hejazi [11], and for solving the linear Helmholtz partial differential equation by Momani and Abuosad [12]. The amplitude and period of limit cycles for a modified Van der Pol oscillator are calculated by He's variational method and the Krylov–Bogoliubov–Mitropolsky method by D'Acunto [13]. VIM is used to construct solitary solutions and compacton-like solutions for nonlinear dispersive equations by He and Wu [14]. The combination of a perturbation method, VIM, method of variation of constants and averaging method to establish an approximate solution of one degree of freedom weakly non-linear systems was proposed by Marinca [15]. Draganescu and Capalnasan [16] applied VIM to a non-linear inelastic model describing the acceleration of the relaxation process in the presence of the vibrations.

In this paper, the variational iteration method is applied to a general non-linear problem, which can be initially approximated with unknown constants. The iterative process is constructed by a general Lagrange multiplier, which can be identified optimally via variational theory. This method is effective and accurate for non-linear problems with approximations converging rapidly to accurate solutions.

2. The iteration procedure

We consider the following nonlinear equation:

$$\ddot{x} + \omega^2 x = f(\Omega t, x, \dot{x}) \tag{2.1}$$

where ω and Ω are positive constants; in general f is assumed to be nonlinear function of Ωt, x and $\dot{x}$, which may be expanded in a Fourier series, and $\dot{x} = \frac{dx}{dt}$. We construct the following iteration formula [7,17]:

$$x_n(t) = x_{n-1}(t) + \int_0^t \lambda(\tau, t)[x''_{n-1}(\tau) + \omega^2 x_{n-1}(\tau) - f(\Omega\tau, \tilde{x}_{n-1}(\tau), \tilde{x}'_{n-1}(\tau))]d\tau \tag{2.2}$$

where $\lambda(\tau, t)$ is called a general Lagrange multiplier, which can be identified optimally via variational theory; $\tilde{x}_{n-1}$ is considered as a restricted variation, i.e. $\delta\tilde{x}_{n-1} = \delta\tilde{x}'_{n-1} = 0$ and $' = \frac{d}{d\tau}$.

The Lagrange multiplier can be readily identified:

$$\lambda(\tau, t) = \frac{1}{\omega}\sin\omega(\tau - t). \tag{2.3}$$

On the other hand, by taking into consideration the identity:

$$\int_0^t \sin\omega(\tau - t)[x''_{n-1}(\tau) + \omega^2 x_{n-1}(\tau)]d\tau = -\omega x_{n-1}(t) + \omega x_{n-1}(0)\cos\omega t + \dot{x}_{n-1}(0)\sin\omega t \tag{2.4}$$

it follows that:

$$x_n(t) = x_{n-1}(0)\cos\omega t + \frac{1}{\omega}\dot{x}_{n-1}(0)\sin\omega t + \frac{1}{\omega}\int_0^t \sin\omega(t - \tau) f(\Omega\tau, x_{n-1}(\tau), x'_{n-1}(\tau))d\tau. \tag{2.5}$$

The Eq. (2.1) may then be written as:

$$\ddot{x} + \Omega^2 x = F(t, \lambda, x, \dot{x}), \qquad \lambda = \Omega^2 - \omega^2, \qquad F(t, \lambda, x, \dot{x}) = \lambda x + f(\Omega t, x, \dot{x}) \tag{2.6}$$

with the initial conditions:

$$x(0) = A, \qquad \dot{x}(0) = 0. \tag{2.7}$$

The left hand side of the Eq. (2.6) has the solution $x = A\cos(\Omega t + \varphi)$, where A and φ are constants, and therefore we try the input of starting function as:

$$x_0 = A_0\cos(\Omega t + \varphi_0). \tag{2.8}$$

According to Ref. [15], we propose the following iteration formula for Eq. (2.6):

$$x_n(t) = A_n\cos(\Omega t + \varphi_n) + \frac{1}{\Omega}\int_0^t \sin\Omega(t - \tau)F(\tau, \lambda, x_{n-1}(\tau), x'_{n-1}(\tau))d\tau, \quad n = 1, 2, \ldots \tag{2.9}$$

where A_n and φ_n are constants and x_0 is given by Eq. (2.8).