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Probabilistic evaluation of spatial distribution of secondary compression by using kriging estimates of geo-layers

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ABSTRACT

This paper presents a procedure for evaluating the spatial uncertainty in the secondary compression (s_s) using a probabilistic method. In order to evaluate the spatial distribution of s_s, the spatial maps of three geo-layers (the thickness and depth of the consolidating layer, the bottom elevation of the reclaimed sandfill) are estimated by using kriging techniques. For all three geo-layers considered in this study, the ordinary kriging is found to give more reliable estimates than the kriging with a trend and simple kriging. It is observed that the coefficients of variation (COVs) of C_α/C_c and $C_c/(1+e_0)$ have similar influences on the COV of s_s. It is also shown that the COV of c_v has less effect on the COV of s_s than the COVs of C_α/C_c and $C_c/(1+e_0)$ although the COV of c_v is larger than that of C_α/C_c and $C_c/(1+e_0)$. The COV of s_s evaluated by considering all the COVs of soil properties is 0.420, which is 1.4–2.7 times larger than that determined by considering the COV of an individual soil property separately. It is observed that the area exceeding a design criterion increases as the COV of $C_\alpha/(1+e_0)$ increases and the probabilistic design criterion (α) decreases. For Songdo New City, the area ratio decreases from 0.47 for α value of 0.05 to 0.04 for α value of 0.45. The design procedure presented in this paper could be used in the decision making process for a geotechnical engineering design.

1. Introduction

Land reclamation on the foreshore of existing coastlines often overlies soft marine clays, which require soil improvement to reduce undesirable long term settlement, i.e., secondary compression (s_s). Such undesirable long term settlement gives rise to a negative skin friction on piles, cracks on pavements, and differential settlement between roads and buildings, etc. Therefore, s_s needs to be predicted to evaluate its effect on the structure at every design and construction stage. The magnitude of s_s should be estimated spatially because the properties, thickness and depth of geo-layers vary irregularly at every location.

The concept of kriging was first introduced by Krige (1951), and mathematical expressions were subsequently developed by Matheron (1962, 1963a, 1963b). In geotechnical engineering, kriging has been used to estimate the spatial distribution of undrained shear strength, SPT N value, liquefaction potential, and high liquefaction probability values, as well as the thickness and depth of geo-layers. Kriging has also been used to model the spatial variation in the results of cone penetration tests, to characterize the spatial variability of soil properties, and to evaluate the quality of site investigations, etc. (Christakos, 1985; Soulie et al., 1990; Chiasson et al., 1995; Jaksa et al., 1997; Parsons and Frost, 2002; Baise et al., 2006; Mendes and Lorandi, 2008; Marache et al., 2009).

The prediction of s_s is made with many uncertainties due to the inherent variability of soil properties. Although the determination of reliable and representative properties for compressible layers is of fundamental importance in the estimation of long term settlement, the nature of soil deposits suggests that these properties should be described by probability distributions (Corotis et al., 1975). A probabilistic method has been frequently used to evaluate the uncertainty of primary consolidation and secondary compression (Corotis et al., 1975; Freeze, 1977; Athanasiou-Grivas and Harr, 1978; Chang, 1985; Hong and Shang, 1998; Zhou et al., 1999).

This paper presents a procedure for evaluating the spatial uncertainty in s_s using a probabilistic method. The spatial distributions of geo-layers are estimated by using kriging techniques such as ordinary kriging, kriging with a trend, and simple kriging. Based on the evaluated statistics of the soil properties, the mean (μ) and standard deviation (σ) of s_s are calculated by using the first order second moment method. With these results, a probabilistic approach is applied to evaluate the uncertainty in s_s while a deterministic method is used to evaluate the spatial distribution of μ of s_s. Finally, a methodology considering the uncertainty in s_s in the geotechnical engineering design process is presented.

2. Description of study site

Songdo New City has been built on a large scale artificial island of 53.4 km^2 area. As shown in Fig. 1a, Songdo New City is located in the

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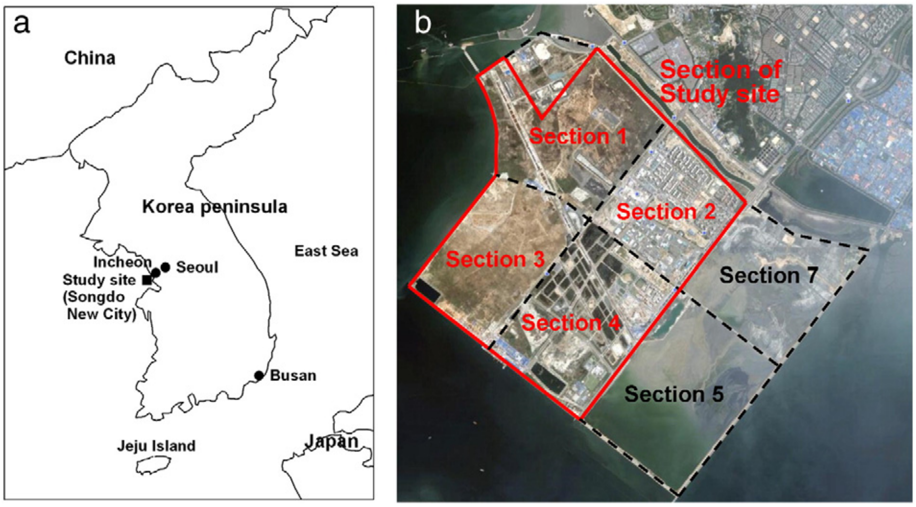

Fig. 1. Study site: (a) location of Songdo New City, and (b) study area (location of Sections 1–4).

western part of Incheon, South Korea (37°21′17″–37°24′33″N, 126°36′40″–126°40′57″E). Sections 1–4 of Songdo New City, which were reclaimed with dredged silty sands between 1997 and 2003, comprise the study area (Fig. 1b). The subsoil of the study area consists of eight distinct units, as shown in Fig. 2: a reclaimed sandfill ($N_{SPT}<15$), upper soft silty clay ($N_{SPT}<6$), upper silty sand ($N_{SPT}>30$), lower medium and stiff silty clay ($N_{SPT}>10$), lower silty sand ($N_{SPT}>40$), weathered soil, weathered rock, and bedrock. Silty sand and silt ($N_{SPT}<10$) are partially observed between the reclaimed sandfill (RSF) and the upper soft silty clay (USSC). N_{SPT} represents the N value of the standard penetration test (SPT). The USSC is a relatively homogeneous layer with average values of N_{SPT} and undrained shear strength of 4 and 30 kPa, respectively. Most of the primary consolidation settlement is expected to occur in the USSC.

3. Geostatistical approach

In a geostatistical approach, the spatial variability and its pattern are usually expressed by an experimental semivariogram, $\gamma(\mathbf{h})$, which is computed as half of the average squared difference between paired data values separated by a vector $\mathbf{h}$:

$$\gamma(\mathbf{h}) = \frac{1}{2N(\mathbf{h})}\sum_{\alpha=1}^{N(\mathbf{h})}[z(\mathbf{u}_\alpha)-z(\mathbf{u}_\alpha+\mathbf{h})]^2 \tag{1}$$

in which $N(\mathbf{h})$ is the number of data pairs within a given class of distance and direction; $z(\mathbf{u}_\alpha)$ is the value at the start or tail of the pair α at the location $\mathbf{u}_\alpha$; and $z(\mathbf{u}_\alpha+\mathbf{h})$ is the corresponding end or head value at a lag of $\mathbf{h}$ from the location $\mathbf{u}_\alpha$. Experimental values for a finite number of separation vectors are obtained using a semivariogram. A theoretical semivariogram model must be fitted to these experimental values to obtain semivariogram values for any possible separation vector used in the kriging interpolation algorithm or kriging matrices. The spherical, exponential, Gaussian, and power models are the most frequently used basic models. The semivariogram stops increasing and fluctuates around a specific value at a specific separation distance. These specific value and separation distance are called the sill and range, respectively. The range denotes the distance between locations beyond which data values appear independent. However, short scale variability, sampling and measurement errors may cause discontinuity at the origin of the variogram. These phenomena are expressed by the nugget effect, and the ratio of the nugget discontinuity to the sill value is referred to as the relative nugget effect (Journel and Huijbregts, 1978; Isaaks and Srivastava, 1989; Cressie, 1991; Deutsch and Journel, 1992; Goovaerts, 1997; Chiles and Delfiner, 1999; Kanevski, 2008).

The purpose of kriging is to estimate the value of a continuous attribute, z, at any unsampled location, $\mathbf{u}$, using the neighboring z-data $\{z(\mathbf{u}_\alpha), \alpha=1,\ldots,n\}$ available over the study area. All kriging estimators are variants of the basic linear regression estimator, $Z^*(\mathbf{u})$, defined as:

$$Z^*(\mathbf{u})-m(\mathbf{u}) = \sum_{\alpha=1}^{n(\mathbf{u})}\lambda_\alpha(\mathbf{u})[Z(\mathbf{u}_\alpha)-m(\mathbf{u}_\alpha)] \tag{2}$$

in which $\lambda_\alpha(\mathbf{u})$ is the weight assigned to $z(\mathbf{u}_\alpha)$, which is a realization of the random variable $Z(\mathbf{u}_\alpha)$; and $n(\mathbf{u})$ is the number of data involved in the estimation; and $m(\mathbf{u})$ and $m(\mathbf{u}_\alpha)$ are the expected values of random variables $Z(\mathbf{u})$ and $Z(\mathbf{u}_\alpha)$. All kriging methods have the same objective of minimizing the error variance, $\sigma_E^2(\mathbf{u})$, under the constraint of unbiasedness of the estimator. That is, $\lambda_\alpha(\mathbf{u})$ are determined by minimizing Eq. (3) under the constraint of Eq. (4) (Deutsch and Journel, 1992; Goovaerts, 1997).

$$\sigma_E^2(\mathbf{u}) = Var\{Z^*(\mathbf{u})-Z(\mathbf{u})\} \tag{3}$$

$$E\{Z^*(\mathbf{u})-Z(\mathbf{u})\} = 0 \tag{4}$$

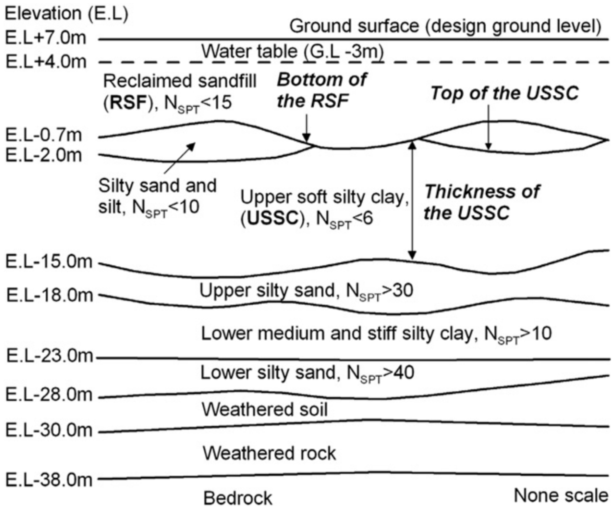

Fig. 2. Schematic soil profile of the study site.