Engineering Geology 114 (2010) 1–9

Contents lists available at ScienceDirect

Engineering Geology

journal homepage: www.elsevier.com/locate/enggeo

A linear regression framework for predicting subsurface geometries and displacement rates in deep-seated, slow-moving landslides

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ARTICLE INFO

Article history:
Received 7 July 2009
Received in revised form 7 March 2010
Accepted 13 March 2010
Available online 20 March 2010

Keywords:
Landslides
Gaussian quadrature
Geometric modeling
Slope stability
Linear regression
Mass movements

ABSTRACT

A new numerical integration/linear regression tool is used to investigate kinematic behavior in deep-seated (sliding surface deeper than 3 m) mass movements. The technique is developed in the context of real inclinometer records and numerical integration using nine inclinometer case histories from four well documented, large landslides: Carrot River, Montebestia, Karya village, and Pietrapertosa. Axial metric, subsurface geometric deformation rates are predicted and compared for four different case studies from the literature: a reactivated, composite, extremely slow earth slide-earthflow; a reactivated composite, extremely slow debris slide-rock fall; an active, composite, very slow earth slide-earth flow; and a reactivated complex, slow earth slide-earth spread. Sensitivity analysis, based on sliding surface depth-to-length (D/L) ratios, show that the mobility of these slow-moving masses is closely dependent on the mode of sliding. The results also show that short term and long term dynamics of slow moving landslides can be captured by geometrical patterns. Because the parameter determined is a geometric property, the technique used in the investigation can be applied in new landslide problems independent of local conditions and triggering mechanisms, within the confines of the stipulated boundary conditions. Hence with this framework also, unrelated landslide problems can be analyzed and compared, as demonstrated herein. Additionally, this approach is useful for two dimensional reconstructions of subsurface displacement and velocity profiles, and thus may act as a precursor to detailed field investigation programs, warning systems and mitigation projects at minimal costs.

1. Introduction

Landslides triggered by seismic loading or abrupt severe climate change can cause loss of lives and substantial property damage. Deep seated landslides (DSL's) pose an even greater challenge because catastrophic failure is often preceded by very slow motion that often goes unnoticed. Unfortunately, there is a general lack of sufficient coherent techniques for characterizing the subsurface dynamics of DSL's, other than field techniques. By carefully monitoring slow movements via field methods such as borehole inclinometers, air photos, surface bench marks, etc., crucial understanding needed for mitigation such as magnitude, direction and depth of movement may be inferred. The major disadvantages of such field investigation programs are that they can be extremely costly and may demand a lengthy time to implement. Catastrophic failures of such large phenomena may occur long before the practitioner has had time to analyze all the data and suggest mitigation measures. Additionally, local conditions and triggering mechanisms can change quickly and often unpredictably further masking needed information. Furthermore, complex geological engineering conditions in DSL's coupled with often several interrelated triggering mechanisms may obscure critical subsurface information such as soil profile, groundwater table and geometry of failure (sliding) surface.

Despite the advantages and availability of several field monitoring techniques, important factors such as landslide deformation/movement mechanisms remain a challenge to determine coherently and in a unified manner, given the irregularity and heterogeneous, non isotropic evolution of landmasses (Nieuwenhuis, 1991; Van Asch et al., 2007).

There are several methods of modeling/characterizing landslide subsurfaces independent of direct field monitoring. Each approach has its own advantages and disadvantages. Desai et al. (1995) presented a method that simulated the velocities and displacements of a creeping slope using parameters obtained from laboratory triaxial measurements. The technique was based on a complex stress–strain behavioral model of soils (the hierarchical single-surface plasticity and viscoplasticity approach), combined with 2D finite elements. The model allowed for elastic, plastic, creep strain, normal stress, and stress path effects. Sitar and MacLaughlin (1997) applied discontinuous deformation analysis to investigate the role of kinematics in landslide behavior. Their numerical method explored the development of displacements with time. The model considers a landslide as a system of individually deformable blocks that move independently. The transient formulation

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doi:10.1016/j.enggeo.2010.03.004

of the problem is based on minimization of potential energy. Mayoraz and Vulliet (2002) used artificial neural networks to predict velocities and displacements in landslides. Inputs to the neural network model included previous velocities, daily precipitation, evaporation, and pore water pressure. The performance of their model was comparable to purely mechanical models. Kilburn and Petley (2003) developed a mathematical expression to characterize accelerating behavior in creeping landslides, leading to catastrophic collapse. Their formulation was based on subsurface deformation processes of landslides, built on earlier observations by Voight (1988).

The main short comings with these approaches include many/difficult variables to specify for analysis, complexity of model formulation, inadequate verification with ground truth data, difficulties in evaluating stress histories, and a general insufficient adoption by practitioners.

The objective of this paper is to combine Gaussian–Legendre quadrature integration with linear regression to investigate the subsurface geometric behavior of several different DSL's. The framework of the method is within the context of monitored surface/near surface displacement rates. It is possible to apply linear regression to relate surface movement rates to subsurface displacements from field instrumentation, such as inclinometer records. The main advantage of this method over exclusively field techniques is that it is an efficient, cost effective way of analyzing landslides. Additionally, the method requires limited assumptions and fewer input parameters compared to typical modeling approaches. This is due to the implicit framework of the mathematical formalism employed; the linear regression applied already takes into account inherent landslide complexities. Consequently, different landslides can be analyzed and compared independent of local parameters. Another advantage is that the technique provides a practical, preliminary tool that can be widely applied before implementing expensive, detailed field investigation programs. Because the technique uses real case data for the initial set up, the resulting mathematical expression is reflective of actual ground truth information. This is useful for sensitivity analysis because data sets from different sources can be investigated, to understand fundamental mechanisms/principles involved.

The major disadvantage of the numerical integration/linear regression technique presented herein is that an initial data set is required that mimics potential applications. This concept is discussed further in Section 2.5. To begin with, the theoretical back ground of the technique is discussed, proceeded by investigative applications in several real examples from the literature. A discussion and conclusions then follow.

2. Background

2.1. Gaussian quadrature

Detailed descriptions of Gaussian quadrature are available in several sources such as Acton (1990), Kincaid and Cheney (2002), among others. Previous applications of Gaussian quadrature in engineering geology include estimation of erosion rate variables (Arndt et al., 2001), terrain corrections in GIS (Hwang et al., 2003), describing groundwater flow in confined aquifers (Yeh et al., 2003), and mass balance calculations in landslides (Kaunda et al., 2008). The following is a brief but essential description of Gaussian quadrature, in light of this study.

Gaussian quadrature is a form of numerical integration where the value of a univariate integral (i.e. an integral involving one variable) may be computed at different pre-selected integration points. When dealing with discrete experimental/field data for example, it is possible to implement a polynomial interpolation through the discrete data or points, which is then integrated. The interpolation will depend on underlying assumptions made and available knowledge about the behavior of the function between the points (Kincaid and Cheney, 2002).

For example, given nodes (or x values of data) $x_0, x_1, \ldots, x_n$ at their respective locations $f(x_i)$ in the interval space $[a,b]$, it is possible to use a polynomial to interpolate the data using a Langrangean form. The Lagrangean form of the polynomial interpolating the data may be written as a summation of products as:

$$p(x) = \sum_{i=0}^{n} f(x_i) l_i(x) \tag{1}$$

where:

$$l_i(x) = \prod_{\substack{j=0 \\ j \neq i}}^{n} \frac{x - x_j}{x_i - x_j} \qquad (0 \leq i \leq n) \tag{2}$$

In brief, the basis of Gaussian quadrature is to obtain the best numerical estimate of an integral by picking optimal *abscissas* x_i at which to evaluate a function $f(x)$. By carefully choosing the location of x_i's and appropriate weighting coefficients, high orders of accuracy can be obtained. If the integration is expressed as:

$$\int_a^b W(x) f(x) dx \approx \sum_{i=1}^{n} w_i f(x_i) \tag{3}$$

then a set of weights, w_i and points, $f(x_i)$ can be found to make the approximation exact. These are obtained by computing the roots of the orthogonal polynomial to the function. A function is said to be orthogonal to another if their scalar product is zero. The roots of the orthogonal polynomial are the *abscissas* of the Gaussian quadrature formula for the same interval and weighting function. The weights can then be determined by solving a system of linear equations formulated from the orthogonal polynomials of different orders: $i = 0, 1, 2 \ldots n$. A much simpler approach is to obtain the abscissas and weights from available mathematical tables such as Abramowitz and Stegun (1972), as demonstrated in this study.

If the weight function $W(x) = 1$, then Eq. (3) becomes the Gauss–Legendre rule for the integration interval $[-1,1]$. This implies that any other integration domain [a, b] must be changed to the interval $[-1,1]$ before applying Gauss–Legendre rule using Eq. (4):

$$\int_a^b f(t) dt = \frac{b-a}{2} \sum_{i=1}^{n} w_i f\left(\frac{b-a}{2} x_i + \frac{a+b}{2}\right) \tag{4}$$

The Gauss–Legendre rule may be applied for different polynomial orders such as $n = 2,3,4,5$ and so on. To make the integration exact, the function $f(x)$ needs to be a polynomial of degree $2n-1$ or less, where "n" represents the degree of the polynomial whose roots are the abscissas, x_i thus:

$$f(x) = 1, x, x^2, \ldots, x^{2n-1} \tag{5}$$

2.2. Volumetric rate of mass flow

In classical integration, the integral of a function represents the area under its curve in Cartesian space. For example, displacement can be represented by the area between a velocity curve and its time axis, and velocity can be represented by the area between the curve of an acceleration function and its time axis. Similarly, the area represented by a velocity function (i.e. displacement rates obtained from measurements such as inclinometer displacement profiles) is equivalent to a very slow, planimetric, *volumetric flow rate per unit width* (Fig. 1). The volumetric rate is said to be "planimetric" because it is derived from a vectoral velocity component, and is two dimensional. It is considered a "flow rate" on the basis of the resulting units from the numerical