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New explicit exact solutions for the generalized coupled Hirota–Satsuma KdV system

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Abstract

In this paper, we study the generalized coupled Hirota–Satsuma KdV system by using the two new improved projective Riccati equations method. As a result, many explicit exact solutions, which contain new solitary wave solutions, periodic wave solutions and combined formal solitary wave solutions and combined formal periodic wave solutions are obtained.

Keywords: Generalized coupled Hirota–Satsuma KdV system; Riccati equations; Solitary wave solution; Periodic wave solution

1. Introduction

In recent years, due to the wide applications of soliton theory in mathematics, physics, chemistry, biology, communications, astrophysics and geophysics, etc., the search for explicit exact solutions, in particular, solitary wave solutions of nonlinear evolution equations (NEEs) has played an important role in the soliton theory. Various effective methods have been developed, such as inverse scattering transformation, Hirota bilinear method, Backlund transformation, Darboux transformation, tanh-function method, extended tanh-function method, sine–cosine method, homogeneous balance method, Jacobian elliptical function expansion method and its generalization, Li group analysis, similarity reduced method, F-expansion method, transformation methods in terms of the Weierstrass elliptical function solutions, and so on.

In 1992, Conte and Musette [1] presented a projective Riccati equation method to seek more new solitary wave solutions to NEEs that can be expressed as polynomial in two elementary functions which satisfy a projective Riccati equation [2]. The method had been applied to find many solitary wave solutions of many equations. In this paper, we will construct two new Riccati equations to generalize the Riccati method. In illustration, we will obtain several new families of exact soliton solutions for the generalized coupled Hirota–Satsuma KdV system.

This paper is arranged as follows. In Section 2, we briefly describe the new extended projected Riccati equation method. In Section 3, several families of solutions to the generalized coupled Hirota–Satsuma KdV system are

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obtained, including new solitary wave solutions and new periodic wave solutions. In Section 4, some conclusions are given.

2. Summary of the new projected Riccati equation method

For a given partial differential equation, say, in two variables x and t

$$P(u, u_t, u_x, u_{xx}, \ldots) = 0. \tag{2.1}$$

We seek the following formal solutions of the given system by a new intermediate transformation:

$$u(x,t) = \sum_{i=0}^{n} A_i f^i(\xi) + \sum_{j=1}^{n} B_j f^{j-1}(\xi) g^j(\xi) \tag{2.2}$$

where $A_0, A_i, B_j, (i, j = 1, 2, \ldots n)$ are constants to be determined later, $\xi = \xi(x,t)$ is arbitrary functions with the variables x and t. The parameter n can be determined by balancing the highest order derivative terms with the nonlinear terms in Eq. (2.1). $f(\xi), g(\xi)$ satisfy the following projective Riccati equations:

$$\text{(I)}\ f'(\xi) = -qf(\xi)g(\xi), \qquad g'(\xi) = q[1 - g^2(\xi) - rf(\xi)], \qquad g^2(\xi) = 1 - 2rf(\xi) + (r^2 + \varepsilon) f^2(\xi) \tag{2.3}$$

where "′" denotes $\frac{\mathrm{d}}{\mathrm{d}\xi}$, $\varepsilon = \pm 1$, r, q are arbitrary constants. It is easy to see that Eq. (2.3) admit the following solutions:

$$f_1(\xi) = \frac{a}{b\cosh(q\xi) + c\sinh(q\xi) + ar}, \qquad g_1(\xi) = \frac{b\sinh(q\xi) + c\cosh(q\xi)}{b\cosh(q\xi) + c\sinh(q\xi) + ar} \tag{2.4}$$

when $\varepsilon = 1$: a, b, c satisfies $c^2 = a^2 + b^2$. When $\varepsilon = -1$: a, b, c satisfies $b^2 = a^2 + c^2$.

$$\text{(II)}\ f'(\xi) = qf(\xi)g(\xi), \qquad g'(\xi) = q[1 + g^2(\xi) - rf(\xi)], \qquad g^2(\xi) = -1 + 2rf(\xi) + (1 - r^2) f^2(\xi). \tag{2.5}$$

Eq. (2.5) have the following solutions:

$$f_2(\xi) = \frac{a}{b\cos(q\xi) + c\sin(q\xi) + ar}, \qquad g_2(\xi) = \frac{b\sin(q\xi) - c\cos(q\xi)}{b\cos(q\xi) + c\sin(q\xi) + ar} \tag{2.6}$$

where a, b, c satisfies $a^2 = b^2 + c^2$. Substituting (2.2) with (2.2) and (2.3) with (2.5) into Eq. (2.1) separately yields a set of differential equations for $f^i(\xi)g^j(\xi)$ $(i, j = 1, 2, \ldots)$. Setting the coefficients of $f^i(\xi)g^j(\xi)$ to zero yields a set of over-determined differential equations (ODEs) in A_0, A_i, B_j, $(i, j = 1, 2, \ldots n)$ and $\xi(x,t)$, solving the ODEs by Mathematica and Wu elimination, we can obtain many exact solutions of Eq. (2.1) according to (2.4) and (2.6).

Obviously, if we choose the special value of a, b, c, q, r in (2.4) and (2.6), then we can get the result of [3–11]. For example, when we choose $b = a = q = 1, c = 0$; $c = a = q = 1, b = 0$ and $q = 1, b = 5, a = 4, c = 3$, then we have $f_{11}(\xi) = \frac{1}{\cosh(\xi)+r}$, $g_{11}(\xi) = \frac{\sinh\xi}{\cosh(\xi)+r}$, $f_{12}(\xi) = \frac{1}{\sinh(\xi)+r}$, $g_{12}(\xi) = \frac{\cosh\xi}{\sinh(\xi)+r}$, $f_{13}(\xi) = \frac{4}{5\cosh(\xi)+3\sinh(\xi)+4r}$, $g_{13}(\xi) = \frac{5\sinh\xi+3\cosh\xi}{5\cosh(\xi)+3\sinh\xi+4r}$, it is the case of [3–7]. If we change the form of (2.4) and (2.6) into sech(ξ), csch(ξ), tanh(ξ), coth(ξ), sec(ξ), csc(ξ), tan(ξ), cot(ξ) type, and choose $b = a = 1$, $q = \sqrt{r_1}, r = \mu, c = 0$; $q = \sqrt{r_1}, r = \mu, b = 0, c = a = 1$ there are only a constant times difference between $f_1(\xi), f_2(\xi), g_1(\xi), g_2(\xi)$ and the σ_i, τ_i $(i = 1, 2, 3, 4)$ in [8,9]. It can be unified completely from the coefficient of $f^i(\xi)g^j(\xi)$ in the assumption form of $u(\xi)$, if we choose $a = 1, q = \sqrt{-pq_1}, b = q_1 l, c = q_1 k$; $a = 1, q = \sqrt{pq_1}, b = q_1 l, c = q_1 k$, so does $f_1(\xi), f_2(\xi), g_1(\xi), g_2(\xi)$ and $f_i(\xi)g_i(\xi)$ $(i = 1, 2)$ in [10] thus (2.4) and (2.6) contain the case of [3–10] completely just the result turns more commonly, the form turns more simply; here many solutions are new.

If we choose $\varepsilon = -1, r = c = 0, b = a, q = 1$ in (2.4), we can obtain the bell-type solitary wave solutions and kink solitary wave solution: $f_{1'}(\xi) = \operatorname{sech}\xi$, $g_{1'}(\xi) = \tanh\xi$, and if we choose $\varepsilon = 1, r = b = 0, c = a, q = 1$, we can obtain singular wave solutions: $f_{1''}(\xi) = \operatorname{csch}\xi$, $g_{1''}(\xi) = \coth\xi$; in common, if choose the special r, c, b, a, q in (2.6), we have $\sec\xi, \csc\xi, \tan\xi, \cot\xi$ type trigonometric function periodic solutions.