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Comparison of indicator kriging, conditional indicator simulation and multiple-point statistics used to model slate deposits

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Abstract

The resources in an ornamental slate deposit can be estimated using geostatistical estimation techniques applied to information collected from drill cores. The result, however, is a smooth approximation that fails to take account of the natural variability in mineralization, which is fundamental to proper design and evaluation of the financial viability of a mining deposit. Geostatistical simulation techniques are more useful in this respect, as they reflect different realizations of the reality and better reflect natural mineral dispersion.

In this work, we evaluate the resources of a slate quarry by comparing the results obtained using two geostatistical techniques—indicator kriging (ik) and sequential indicator simulation (sisim)—with the results obtained using the single normal equation simulation (snesim) technique based on multiple-point statistics (mps), analyzing their usefulness in evaluating the financial risk derive from uncertainty in regard to knowledge of the deposit. Our results indicate that although the multiple-point statistics approach produces models that are closer to reality than the models produced by the geostatistical techniques, the simulation relies in part on information obtained via indicator kriging.

Keywords: Geostatistics; Indicator kriging; Conditional simulation; Multiple-point statistics; Mining resource evaluation

1. Introduction

The mining of ornamental rocks, and especially slate for roofing, has undergone spectacular development during the last 30 years in Spain, a fact which has placed this country first among the world's slate producers. This has brought about a considerable improvement in both productive processes and the machinery used, but there are a number of technical factors that have not yet been satisfactorily resolved. One example is the exploration and evaluation of slate deposits for mining purposes.

Slate is a fine-grained, homogeneous, metamorphic rock derived from an original shale-type sedimentary rock composed of clay through low-grade regional metamorphism. It is mainly composed of quartz and muscovite or illite, often along with biotite, chlorite, hematite and pyrite, and less frequently, apatite, graphite, kaolin, magnetite, tourmaline, zircon and feldspar.

As methods for slate quarrying, we can distinguish clearly between the more frequently utilized surface mining method, and the underground method that is gradually becoming more popular (Taboada et al., 2006a). Slate is commonly used as a roofing and flooring material and for electrical insulation.

Geological investigation of a slate deposit is mainly based on continuous drill cores, since surface alterations impede surface sampling of fresh rock. The cores are used to study the sedimentary, metamorphic and structural characteristics that define the orebody. The orebody typically contains a mass of useful slate (which is defined, as we shall see subsequently, as the equivalent of mineral grade) and areas of spoil, the latter mainly a consequence of structural discontinuities in the orebody. In slate mining as in the rest of mining, however, all knowledge is inevitably linked to uncertainty, and this is represented as risk when a mining deposit is being assessed. In order to obtain the necessary decision-making

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elements for each of the developmental phases of the mining project, the mining company has to quantitatively calculate a value for each parameter that intervenes in the mining evaluation along with the uncertainty associated with these values. This uncertainty has a significant bearing on the mining company's final decision as to whether to assume the risk of allocating human and financial resources to the development of a mining project.

The problem of grade uncertainty and risk effects in open-pit design is addressed by Dimitrakopoulos et al. (2002), who use generalized sequential Gaussian simulation and direct block simulation techniques to model and evaluate the financial risk implied by a typical, disseminated, low-grade, epithermal, quartz breccia-type gold deposit. In this article, we tackle the same problem for slate deposits, although taking a somewhat different approach. We estimate useful slate reserves using indicator kriging and compare the results with those obtained using conditional simulation techniques and multiple-point statistics (Guardiano and Srivastava, 1993).

2. Roofing slate properties: Useful slate and saleable slate

Only part of the slate in a quarry can be exploited as roofing slate, as slate for this purpose must have certain physical, chemical and aesthetic properties and must not be affected by structures that interrupt the cleavage direction. This slate is referred to as *useful slate*.

From a general perspective, the usefulness of slate for roofing is determined from samples extracted from quarry fronts which are laboratory tested for compliance with the Spanish UNE standard 12326-1: 2005 (AENOR, 2005). These tests analyze slate samples in terms of a range of physical and chemical properties such as fissility, impermeability, carbonate content, colour, alterability, inclusions, homogeneity of grain size, and roughness. (See Taboada et al., 2006b) for a more detailed description of these properties).

The test results combined with data from continuous drill cores enable experts to determine which parts of the drill cores represent useful slate. Thus, the expert divides the cores into 1-m long stretches and assigns them to Class 1 if classified as useful slate and Class 0 if classified as non-useful slate. The choice of 1-m lengths is justified by the fact that mining requires a minimum block thickness of 50 cm; consequently, as thickness is given by the normal direction to the cleavage plane that dips 60°, the support can be considered to be representative of the minimum block size in that direction. As will be demonstrated in Section 5.1 below, using indicator kriging will enable us to estimate both the percentage of stretches belonging to Class 1 within the blocks into which the quarry will be discretized and the percentage of useful slate contained in this support (in other words, the mineral grade for the block). In this way gross slate reserves can be estimated and mining can be designed optimally (Bastante et al., 2005).

Of the useful slate indicated as such by the drill core sampling, only part can be recovered from the quarry in the form of blocks. Only a small proportion of this recovered slate is saleable, moreover, and only after the slate has undergone various mechanical operations in the slate transformation processing plant. This slate is referred to as finished or saleable slate.

3. Methodologies used to evaluate slate resources

In our research, we used different techniques to evaluate useful slate reserves from the information provided by cores made in a slate quarry. Below we summarize the theory underlying each of the techniques and subsequently compare and discuss the results obtained for each method.

3.1. Indicator kriging

The basic approach taken by predictive statistics is to model the uncertainty associated with an unsampled value, $z(\mathbf{u})$, with $\mathbf{u}$ as the coordinate location vector, as a random variable (RV), $Z(\mathbf{u})$, the probability distribution of which characterizes the uncertainty about $z(\mathbf{u})$ (Deutsch and Journel, 1998). Categorical variables such as rock types can be effectively modelled by RVs. Let $Z(\mathbf{u})$ be a categorical RV that can take either of 2 outcome values $k=\{k1,\ k2\}$. In the particular case to be analyzed here, $z(\mathbf{u})$ is the variable stretch of a metre of slate centered on $\mathbf{u}$; $Z(\mathbf{u})$ is the random function that models $z(\mathbf{u})$; and $(k1, k2)$ corresponds to the categories (Class 1, Class 0) or useful/non-useful slate. The conditional probability density function is:

$$f(\mathbf{u};k|(n)) = \mathrm{Prob}\{Z(\mathbf{u}) \in \text{category } k|(n)\}$$

(n) consisting of n neighbouring data values $z(\mathbf{u}_\alpha)$, $\alpha=1,\ldots,n$.

The essence of the indicator approach is the binomial coding of $Z(\mathbf{u})$ into either 1 or 0 depending upon its relationship to a categorical class. By considering binary indicator transforms of $Z(\mathbf{u})$ defined as:

$$I(\mathbf{u};k) = \left\{ \begin{array}{l} 1,\ \text{if } Z(\mathbf{u}) \in k \\ 0,\ \text{otherwise} \end{array} \right\}$$

the probability that a category k prevails at $\mathbf{u}$ can be expressed as:

$$E\{I(\mathbf{u};k|(n))\} = \mathrm{Prob}\{(\mathbf{u};k) = 1|(n)\} = f(\mathbf{u};k|(n)).$$

Applying the kriging algorithm to indicator data we obtain the least-squares estimate of its conditional expectation, $f^*(\mathbf{u};k|(n)$. Note that the bivariate (two-point) distribution of 2 RV's $Z(\mathbf{u}_1)$, $Z(\mathbf{u}_2)$, appears as the non-centered covariance of the indicator variables:

$$\begin{aligned} f(\mathbf{u}_1,\mathbf{u}_2;k_1,k_2) &= \mathrm{Prob}\{Z(\mathbf{u}_1) = k_1,\ Z(\mathbf{u}_2) = k_2\} \\ &= E\{I(\mathbf{u}_1;k_1)I(\mathbf{u}_2;k_2)\} \end{aligned}$$

and the indicator variogram function measures how often two locations that are a vector $\mathbf{h}$ apart belong to different categories. Indicators are useful for characterizing the spatial variability of categorical variables as the ranges and shapes of the directional indicator semivariograms that reflect geometric patterns (Goovaerts, 1997).

3.2. Sequential indicator simulation

Estimating reserves by means of kriging produces a smoothed representation of the reality, and this typically underestimates real data dispersion (Journel, 1974). Since geostatistics considers the