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Extension of Coefficients for (n, k, m) Convolutional-Code-Based Packet Loss Recovery

MASAYUKI ARAI, SATOSHI FUKUMOTO AND KAZUHIKO IWASAKI
Graduate School of Engineering
Tokyo Metropolitan University, 1-2, Minami-osawa, Hachioji, Tokyo 192–0397, Japan
<arai><fukumoto><iwasaki>@eei.metro-u.ac.jp

Abstract—This paper discusses a new idea for improving the recovery of (n, k, m) convolutional-code-based packet-level FEC. We extend the coefficients in the generator matrix to the elements on $GF(2^8)$. Each byte in the redundant packets is generated independently, assuming the bytes are the elements on $GF(2^8)$. The postreconstruction receiving rate is analytically derived from the necessary conditions for recovery. Moreover, the ability to recover lost packets is estimated based on analysis, and we demonstrate that the extension of coefficients improves the number of recovered packets more than the conventional method where coefficients are 0 or 1.

Keywords—Convolutional code, Forward error correction, Packet loss recovery, Postreconstruction receiving rate.

1. INTRODUCTION

Packet-level forward error correction (FEC) is one of the major strategies that improves the dependability of communication over the Internet. Recoverability of packet losses using FEC depends on applied coding schemes, and, of course, improving recoverability is an underlying problem. This paper discusses our new approach to increasing the recoverability of convolutional-code-based packet-level FEC.

Automatic repeat request (ARQ) and FEC are the two basic categories for packet recovery. ARQ and FEC take different recovery processes, and then each has the suitable type of applications. While ARQ is based on the retransmission-based approach, FEC recovers lost packets using received information packets as well as received redundant packets, which are generated from information packets and sent along with them. The recovery process does not require the retransmission of lost packets. Thus, FEC-based approaches are suitable for applications such as real-time transmission that require the recovery of as many packets as possible within a limited time.

FEC needs to generate redundant packets by applying an appropriate coding/decoding scheme. Recoverability, therefore, or the ability to recover lost packets, depends on the coding schemes.

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While the application of Reed-Solomon (RS) codes to erasure channels is a well-known scheme [1,2], we have proposed the application of (n, k, m) convolutional code to packet-level erasure channel [3,4]. Convolutional codes have been studied as error detecting and correcting codes [5,6]. Massey reported the application of q-ary convolutional code to the erasure channel [7]. However, this scheme utilizes q Viterbi decoders independently, and thus it is not realistic to apply to packet recovery over the Internet where the length of packets sometimes exceeds 10000 bits. Our approach recovers lost packets by solving equation systems derived from the positions of losses without using Viterbi decoders. In [3], we derived the conditions that the lost packets can be recovered, and evaluated the ability to recover packets using simulations. Under some conditions, the proposed method showed superior ability of recovery to the RS-code-based approach. In [4], we applied it to reliable multicast communication, and analyzed the number of transmissions and transmitted packets.

In the previous works mentioned above, we assumed that the bits in a redundant packet were generated in parallel, that is, the coefficients of generator matrices were elements on $GF(2)$. This assumption sometimes causes some packets to remain unrecovered because simultaneous equations for recovery do not have a unique solution. Then, by extending the coefficients to elements on larger Garois fields, we can expect to improve recoverability even further.

In this paper, we extend the coefficients in the generator matrix of the proposed method to elements on $GF(2^8)$. We discuss encoding/decoding schemes and evaluate recovery with our new approach. The analysis derives an equation to estimate the postreconstruction receiving rate (PRRR), i.e., the probability that a packet is able to be received or be recovered. Using analytical results enables us to readily calculate the PRRRs for various parameters, because we do not have to spend a long time on calculation such as that involved in simulations.

This paper is organized as follows. Section 2 describes the encoding and decoding schemes. In Section 3, we analytically derive a formula to evaluate recovery of lost packets. We prove the effectiveness of our method through numerical examples, based on the results of the analysis. Section 4 summarizes our findings.

2. ENCODING AND DECODING SCHEME

2.1. Encoding Scheme

This is an overview of the encoding scheme, which applies (n, k, m) convolutional codes to generate redundant packets.

The sequence of information packets that the sender generates is divided into groups. Each group consists of k information packets. Let $\mathbf{u}_i = [u_{i,1} u_{i,2} \cdots u_{i,k}]$ denote the i^{th} group, where $u_{i,j}$ expresses an information packet. In the following, we assume that each packet has a fixed length of a q-bits.

The group of n packets $\mathbf{v}_i = [v_{i,1} v_{i,2} \cdots v_{i,n}]$, which we call the code group, is generated from each group of information packets, $\mathbf{u}_i$, as follows:

$$\mathbf{v}_i = \mathbf{u}_i \cdot \mathbf{Q}(D), \tag{1}$$

where $\mathbf{Q}(D)$ is the $k \times n$ generator matrix whose elements contain the delay operator D. The code being applied here is a systematic code. When $\mathbf{p}_i = [p_{i,1} \cdots p_{i,n-k}]$ denotes $n - k$ redundant packets in $\mathbf{v}_i$, the following equation holds:

$$v_{i,j} = \begin{cases} u_{i,j}, & j \leq k, \\ p_{i,j-k}, & j > k. \end{cases} \tag{2}$$