Computers & Operations Research 76 (2016) 22–32

Contents lists available at ScienceDirect

Computers & Operations Research

journal homepage: www.elsevier.com/locate/caor

An efficient alternative to the exact evaluation of the quickest path flow network reliability problem

M. El Khadiri [a,*], W.-C. Yeh [b]

[a] Department Gestion Logistique & Transport, IUT of Saint-Nazaire, University of Nantes, 58, rue Michel Ange, 44606 Saint-Nazaire Cedex, France
[b] Integration and Collaboration Laboratory, Department of Industrial Engineering and Engineering Management, National Tsing Hua University, Hsinchu, Taiwan

ARTICLE INFO

Article history:
Received 21 September 2015
Received in revised form
21 March 2016
Accepted 15 June 2016
Available online 18 June 2016

Keywords:
Arc capacity
Arc lead time
Flow network
Inclusion–Exclusion
Minimal Path
Monte–Carlo simulation
Path capacity
Path lead time
Path transmission time
Quickest path
Reliability

ABSTRACT

In this paper we consider the evaluation of the probability that a stochastic flow network allows the transmission of a given amount of flow through one path, connecting the source and the sink node, within a fixed amount of time. This problem, called the quickest path flow network reliability problem, belongs to the NP-hard family. This implies that no polynomial algorithm is known for solving it exactly in a CPU runtime bounded by a polynomial function of the network size. As an alternative, we propose to perform estimations by a Monte–Carlo simulation method. We illustrate that the proposed tool evaluates, with high precision and within small CPU runtime, configurations which cannot be handled, in reasonable CPU runtime, by means of a well-known exact method.

1. Introduction

The quickest path problem consists of identifying a $s-t$ path in a network to transmit an amount d of flow from s to t with minimal transmission time, when each arc a_i of the network is assigned a lead time value l_i and a capacity value c_i. This optimization problem has attracted great attention of researchers due to its usefulness in a wide range flow network applications. It was first proposed to find the fasted route for convoy-type traffic in flow-rate constrained network [24] and was used later in communication networks where nodes represent transmitters/receivers and arcs communication channels [11]. Polynomial time algorithms for this problem and for ranking the first K quickest paths were provided and analyzed in [4,9,10,23,25,26,28]. The all-pairs quickest path problem was solved in [8,17].

When arcs may fail randomly, each path has a functioning probability. In this case, it is of interest to consider the reliabilities of quickest paths. Polynomial time algorithms have been proposed for the quickest most-reliable and the most-reliable quickest path problems in [32] and for all-pairs quickest most-reliable and all-pairs most-reliable quickest path problems in [1]. In [30], pseudo polynomial exact methods and fully polynomial approximation methods were proposed to find a quickest path among those with at least a prefixed reliability. The latter is a generalization of the most reliable quickest path problem solved by reducing it to the restricted quickest path case where arcs are assigned costs and the objective is to find a quickest path among those not exceeding a threshold cost. For the most reliable quickest path problem, the reader can also see the work in [5] where two approaches have been compared with respect to the required CPU runtime.

In many real-world flow networks, the capacities of arcs may have several possible states due to the failure, the maintenance, the traffic or other conditions. In order to take into account this behavior, each arc a_i is assigned a random capacity variable C_i, instead of a deterministic value c_i. For such type of network, called a multi-state flow network, the minimum transmission time of an amount of flow through a $s-t$ path is also a random variable. For the case when arc capacities are independent random variables, the quickest path problem was extended in [18] to the computation of the probability that the network allows the transmission of d units of flow from s to t through a $s-t$ path within τ units of

* Corresponding author.
E-mail addresses: mohamed.el-khadiri@univ-nantes.fr (M. El Khadiri), yeh@ieee.org (W.-C. Yeh).

http://dx.doi.org/10.1016/j.cor.2016.06.010
0305-0548/

time. As this event is equivalent to the event “the transmission time through a quickest path from s to t is smaller than or equal to τ”, this problem is called the quickest path flow network reliability problem. In [18,34] exact methods have been devoted to its evaluation. Unfortunately, this problem is NP-hard. Such type of limitation leads researchers to consider Monte–Carlo techniques which give estimates with associated confidence intervals. These techniques, widely used for evaluating connectivity reliability measures (see [7] for many references) and maximal flow network reliability measures [3,13,27], was not considered for the quickest path flow network reliability case. As a consequence, our objective in this paper is to provide a Monte–Carlo algorithm dedicated to this problem and to illustrate its interest when compared to the exact approach.

Before presenting the organization of this paper, it is worth noticing that other reliability measures exist. For instance, if the amount d of flow can be split into 2 parts, Lin proposes to evaluate the probability of the transmission simultaneously through 2 prefixed disjoint $s-t$ paths, within τ units of time [21]. Subsequently, the optimal pair of $s-t$ paths with highest probability is identified by considering each pair of disjoint paths. Due to the high CPU runtime to determine the optimal pair, in [16] the authors propose to find a pair of $s-t$ paths such that the probability of successful flow transmission is not smaller than a given reliability threshold. In addition to the time threshold τ, reliability measures which take into account budget constraint and the transmission through k fixed disjoint $s-t$ paths are presented and computed in [19,20].

The remainder of the paper is organized as follows. In the following section we introduce general notation, definitions and assumptions. In Section 3, we briefly recall the exact method provided in [18]. In Section 4, we propose a Monte–Carlo method for estimating the reliability measure under consideration. Section 5 contains some illustrations which show the interest of the proposed approach. Finally, we present conclusions and future work in Section 6.

2. Notations and problem formulation

Let us denote by $\mathcal{N}=\left(\mathcal{V},\mathcal{E},\mathcal{L},\vec{C},s,t,d,\tau\right)$ the stochastic flow network where

n is the number of nodes,
m is the number of arcs,
$\mathcal{V}=\{v_1,\ldots,v_n\}$ is the set of nodes,
$\mathcal{E}=\{a_1,\ldots,a_m\}$ is the set of arcs,
$\mathcal{L}=\{l_1,\ldots,l_m\}$ is the set of lead time with l_i is the lead time of the arc a_i,
$\vec{C}=\left(C_1,\ldots,C_m\right)$ is the random capacity vector with C_i is the random capacity of the arc a_i,
$\Omega_i=\{c_{i_1},c_{i_2},\ldots,c_{i_{n_i}}\}$ is the state space of C_i,
n_i is the cardinality of Ω_i,
r_{i_j} is the probability that C_i takes value c_{i_j},
c_i^+ is the maximal capacity of a_i,
s is the source node,
t is the sink node,
d is the amount of flow which we have to send from node s to node t,
τ is the maximal transmission time.

The stochastic flow network $\mathcal{N}=\left(\mathcal{V},\mathcal{E},\mathcal{L},\vec{C},s,t,d,\tau\right)$ is assumed to fulfill the following conditions:

nodes are perfectly reliable,
the probability mass function of each arc random capacity variable is known,
arcs random capacity variables are statistically independent (s-independent),
elements of $\Omega_i=\{c_{i_1},c_{i_2},\ldots,c_{i_{n_i}}\}$ are integers which verify $0\le c_{i_1}<c_{i_2}<\cdots<c_{i_{n_i}}=c_i^+$,
flow is transmitted through one $s-t$ path.

In such network, we consider the evaluation of the probability $\mathcal{R}$ that the network allows to send d units of flow from s to t within τ units of time. Let us fix an element $\vec{c}=(c_1,\ldots,c_m)$ of Ω, which denotes the state space of the random capacity vector $\vec{C}\left(\Omega=\otimes_{i=1}^m\Omega_i\right)$. When $\vec{C}=\vec{c}$, the transmission time $\tau(P,d,\vec{c})$ through a $s-t$ path P depends on its lead time, deduced from the lead times of its arcs by the formula

$$L(P)=\sum_{a_l \text{ belongs to } P} l_i \tag{1}$$

and on its capacity $B(P,\vec{c})$ which is equal to the smallest capacity among the capacities of its arcs when $\vec{C}=\vec{c}$:

$$B(P,\vec{c})=Min\{c_i,\ a_i \text{ belongs to } P\}. \tag{2}$$

More precisely, we have

$$\tau(P,d,\vec{c})=\begin{cases} L(P)+\left\lceil\dfrac{d}{B(P,\vec{c})}\right\rceil & \text{if } B(P,\vec{c})\neq 0\\ +\infty & \text{otherwise}\end{cases} \tag{3}$$

where $\lceil x\rceil$ is the smallest integer such that $\lceil x\rceil\ge x$.

A capacity vector $\vec{c}\in\Omega$ is an operational state if and only if there is at least one $s-t$ path P in the network such that $\tau(P,d,\vec{c})\le\tau$ and the reliability parameter $\mathcal{R}$ is

$$\mathcal{R}=\sum_{\vec{c}\in\Omega^+}\Pr\left\{\vec{C}=\vec{c}\right\} \tag{4}$$

where $\Omega^+\subseteq\Omega$ is the set of all operational capacity vectors. As the capacity random variables are statistically independent we have

$$\Pr\left\{\vec{C}=\vec{c}\right\}=\prod_{i=1}^m\Pr\{C_i=c_i\}.$$

3. Exact evaluation of $\mathcal{R}$ by the method proposed in [18]

To compute $\mathcal{R}$ by means of Formula (4) may lead to prohibitive CPU runtime when the number of arcs and/or cardinalities of Ω_i are high. Indeed, the number of states to consider in order to identify the operational states is the cardinality of Ω which is equal to $n_1\times n_2\times\ldots\times n_m$, where n_i is the number of possible capacities of the arc a_i. The method proposed in [18] avoids this formula by exploiting the set of $s-t$ paths. In this section, we recall briefly this method and we illustrate its application on a small flow network. Notations and definitions introduced here will be useful in the later section where we propose a Monte–Carlo algorithm dedicated to the estimate of $\mathcal{R}$.

Let us consider a $s-t$ path P. The maximal possible capacity $c^+(P)$ which P may offer, depends on the maximal capacities of its arcs:

$$c^+(P)=Min\{c_i^+,\ a_i \text{ belongs to } P\}. \tag{5}$$

Based on Formula (3), the smallest integer capacity $c^*(P,d,\tau)$, which must be offered by P for the transmission of the amount d of data within τ units of time, is