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Inventory management for new products with triangularly distributed demand and lead-time

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ABSTRACT

This paper proposes a computational methodology to deal with the inventory management of new products by using the triangular distribution for both demand per unit time and lead-time. The distribution for demand during lead-time (or lead-time demand) corresponds to the sum of demands per unit time, which is difficult to obtain. We consider the triangular distribution because it is useful when a distribution is unknown due to data unavailability or problems to collect them. We provide an approach to estimate the probability density function of the unknown lead-time demand distribution and use it to establish the suitable inventory model for new products by optimizing the associated costs. We evaluate the performance of the proposed methodology with simulated and real-world demand data. This methodology may be a decision support tool for managers dealing with the measurement of demand uncertainty in new products.

1. Introduction

Studying uncertainty of demand during lead-time or lead-time demand (LTD) is a key aspect not only for retailing and manufacturing, but also for supply chain planning [9]. This uncertainty is present because demand per unit time (DPUT) and lead-time (LT) usually occur in a stochastic fashion. Therefore, DPUT, LT and LTD are random variables (RVs) following statistical distributions, which can be characterized by their corresponding probability density functions (PDFs).

We assume that the LTD is a random sum until the LT of independent DPUTs, that is, the corresponding DPUT time series is uncorrelated. The PDF of the LTD distribution is useful to determine the components of probabilistic inventory models. A model that is often used for inventory supply planning is the (Q, r) model, which is based on the order quantity or lot size (Q) and reorder point (r). Note that Q corresponds to the quantity to be ordered when the stock achieves a certain amount of products r. The reorder point often includes a safety stock (SS) corresponding to a buffer stock used to mitigate the risk of a stock-out. The model components Q and r must be determined to minimize the total cost of the inventory management. Such a cost is function of the holding, ordering and shortage costs. When calculating the reorder point for a fixed service level, the LTD PDF is used. If the LTD distribution is unknown, this PDF can be approximated by some suitable approach. We simultaneously optimize Q and r as detailed in Section 2.4; see also [33].

The Gaussian (or normal) distribution is often employed to describe the RVs DPUT, LT and LTD due to its attractive properties. However, assuming normality is not always suitable to model these RVs [23,29,36]. When historical DPUT data for a single-product are available, a pool of non-normal distributions can be considered as candidates for modeling these data. To obtain the inventory management model, the suitable DPUT distribution must be selected by standard goodness-of-fit (GOF) methods [1]. Nevertheless, there are cases where the associated LTD distribution is difficult to obtain. Under such circumstances, empirical distributions generated from raw data may be helpful for decision making [35]. In the case of new products, modeling DPUT, LT and LTD is difficult because historical data are unavailable, but business decisions must be made prior to the availability of these data [14,22]. Ref. [5] studied inventory models with a lognormal DPUT distribution and indicated the LTD distribution under different DPUT and LT distributions.

Demand uncertainty for new products has been handled by learning-based and non-learning-based approaches [7]. Under learning-based approaches, multiple production or purchasing commitments are decided first in such a way that sales data should be further obtained to update the demand forecasts and,

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then, to review these production/purchasing commitments [3,6,28]. Under non-learning-based approaches, [8] used judgmental forecasts to establish demand uncertainty, whereas [36] employed a uniform (UNI) distribution in that same case. Demand uncertainty of new products could also benefit from other non-learning-based approaches, such as approximations of untractable LTD distributions by considering tractable DPUT and LT distributions.

The triangular (TRI) distribution is tractable and known to be useful when data are unavailable, difficult to obtain or expensive to collect [10]. This distribution can be used for involving managers in the analytical process by considering their subjective estimates of the minimum, most likely (mode) and maximum values. According to [15], the TRI distribution has the advantage of being intuitively plausible to practitioners. However, despite its long history dating back to [32], its recognition as a user-friendly tool is more recent [19]. Assuming triangularity may help managers in dealing with new products, which have no historical data, and therefore, offer no possibility of establishing analogies with similar products. Based on this assumption, managers may decide the first lot size to be ordered and the reorder point.

The objective of the present paper is to propose a novel computational methodology for inventory management of new products. Specifically, we consider TRI distributions for modeling both DPUT and LT. In this case, the LTD distribution is unknown. Based on the (Q, r) inventory model, we need the LTD PDF to determine the components Q and r that minimize the expected inventory total cost. We provide an approach to estimate the actual PDF of the unknown LTD distribution obtained from triangularly distributed DPUT and LT by using polynomials and a mixture of truncated exponentials (MTEs). We evaluate the quality of the proposed approach with the Kullback–Leibler (KL) divergence [20] using the kernel non-parametric method to estimate the unknown LTD PDF such as [21]. Then, we employ the approach to the unknown PDF for establishing a computational solution to the (Q, r) inventory model for new products optimizing the associated costs. Components Q and r are found by using the bisection method on the partial derivatives of the total cost function with expected shortages per cycle, which are studied under different scenarios [13]. Managerial implications for inventory decision-making are also addressed.

Section 2 proposes the novel methodology; Section 3 discusses a computational framework for this methodology and conducts simulations to evaluate its performance; Section 4 illustrates its potential applications with real-world data; and Section 5 provides our conclusions and possible future works.

2. Methodology

In this section, we propose a methodology for inventory management of new products. In Section 2.1, we present a background on the TRI distribution, which is helpful to model both DPUT and LT, when their distributions are unknown due to data unavailability or difficulties to collect them. In Section 2.2, we provide some details on LTD distributions obtained from the sum of independent RVs, which are useful for determining the LTD PDF. In Section 2.4, we approximate the unknown LTD PDF resulting from triangularly distributed DPUT and LT by using polynomials and MTEs. The LTD PDF is needed to determine Q and r when minimizing the inventory cost. In Section 2.5, we define the KL divergence to evaluate the quality of the proposed approximations in relation to an actual PDF, which is obtained with the kernel method described in Section 2.3. At last, in Section 2.6, we compute an analytical solution of the (Q, r) model considering the polynomial approximation for the LTD PDF.

2.1. Triangular distribution

Let T be a continuous RV following a TRI distribution with parameters $a, b, c \in \mathbb{R}$, where a and b are the minimum and maximum values of T, respectively, and c is the mode of the distribution. This is denoted by $T \sim \text{TRI}(a,b,c)$. Then, the PDF, cumulative distribution function (CDF) and quantile function (QF) of T are, respectively, given by

$$f_T(t) = \frac{\mathrm{d}F_T(t)}{\mathrm{d}t} = \begin{cases} \dfrac{2(t-a)}{(b-a)(c-a)} & \text{if } a \le t \le c; \\ \dfrac{2(b-t)}{(b-a)(b-c)} & \text{if } c \le t \le b; \\ 0 & \text{otherwise;} \end{cases}$$

$$F_T(t) = \mathrm{P}(T \le t) = \int_{-\infty}^{t} f_T(v)\mathrm{d}v = \begin{cases} 0 & \text{if } t < a; \\ \dfrac{(c-a)}{(c-b)}\dfrac{(t-a)^2}{(c-a)^2}, & \text{if } a \le t \le c; \\ 1 - \dfrac{(b-c)}{(b-a)}\dfrac{(b-t)^2}{(b-c)^2}, & \text{if } c \le t \le b; \\ 1 & \text{if } t > b; \end{cases}$$

$$t(q) = F_T^{-1}(q) = \begin{cases} a + \sqrt{q(c-a)(b-a)} & \text{if } 0 \le q \le (c-a)/(b-a); \\ b - \sqrt{(1-q)(b-c)(b-a)} & \text{if } (c-a)/(b-a) \le q \le 1. \end{cases} \quad (1)$$

A random number generator for $T \sim \text{TRI}(a,b,c)$ is provided in Algorithm 1 based on (1).

Algorithm 1. Random number generator for the TRI distribution.

1: Generate a uniform value u from $U \sim \text{UNI}(0,1)$.
2: Set values for a, b and c of $T \sim \text{TRI}(a,b,c)$;
3: Compute a random number $t = t_1$ or $t = t_2$ from $T \sim \text{TRI}(a,b,c)$ using (1), that is,
 3.1: If $0 \le u \le (c-a)/(b-a)$, then $t_1 = a + \sqrt{u(c-a)(b-a)}$;
 3.2: Else $t_2 = b - \sqrt{(1-u)(b-c)(b-a)}$;
4: Repeat Steps 1–3 until the required number of LTD observations has been generated.

The mean and variance of $T \sim \text{TRI}(a,b,c)$ are, respectively,

$$\lambda = \mathrm{E}(T) = \frac{a+b+c}{3}, \quad \sigma^2 = \mathrm{Var}(T) = \frac{(b-a)^2}{18}\left(1 - \frac{(c-a)(b-c)}{(b-a)^2}\right). \quad (2)$$

2.2. Demand distribution during lead-time

Let X be a RV corresponding to the DPUT, which has mean $\mathrm{E}(X) = \lambda_X$ and variance $\mathrm{Var}(X) = \sigma_X^2$. In addition, let the RV L be the LT between the ordering of a product and its delivery, which has mean $\mathrm{E}(L) = \lambda_L$ and variance $\mathrm{Var}(L) = \sigma_L^2$. Furthermore, L is assumed to be independent from each element of the sequence of independent identically distributed RVs $\{X_1, X_2, \ldots, X_L\}$ obtained from the RV X. Moreover, assume that orders do not cross [12]. Therefore, the LTD for a product is the random sum given by

$$Y = X_1 + X_2 + \cdots + X_L, \quad (3)$$

with PDF $f_Y(\cdot)$ defined on $[0, \infty)$ (non-negative support), CDF

$$F_Y(y) = \int_0^y f_Y(v)\,\mathrm{d}v, \quad (4)$$

and QF $y(q) = F_Y^{-1}(q)$, for $0 < q < 1$. The expectation and variance of Y are, respectively, expressed as

$$\mathrm{E}(Y) = \mathrm{E}(L)\mathrm{E}(X) = \lambda_L \lambda_X, \quad (5)$$

$$\mathrm{Var}(Y) = \mathrm{Var}(L)(\mathrm{E}(X))^2 + \mathrm{E}(L)\mathrm{Var}(X) = \sigma_L^2 \lambda_X^2 + \lambda_L \sigma_X^2. \quad (6)$$

Note that, in general, the LT and DPUT can be modeled by any