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Robust newsvendor problem with autoregressive demand

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ABSTRACT

This paper explores the single-item newsvendor problem under a novel setting which combines temporal dependence and tractable robust optimization. First, the demand is modeled as a time series which follows an autoregressive process $AR(p)$, $p \geq 1$. Second, a robust approach to maximize the worst-case revenue is proposed: a robust distribution-free autoregressive method for the newsvendor problem, which copes with non-stationary time series, is formulated. A closed-form expression for the optimal solution is found for $p=1$; for the remaining values of p, the problem is expressed as a nonlinear convex optimization program, to be solved numerically. The optimal solution under the robust method is compared with those obtained under three versions of the classic approach, in which either the demand distribution is unknown, and autocorrelation is neglected, or it is assumed to follow an $AR(p)$ process with normal error terms. Numerical experiments show that our proposal usually outperforms the previous benchmarks, not only with regard to robustness, but also in terms of the average revenue. Extensions to multiperiod and multiproduct models are also discussed.

1. Introduction

The single-period problem (SPP), also known as the newsvendor problem, is a simple yet rich inventory model which has been widely studied in the Operations Research field due to its versatility and applicability to many business decision problems, in fields such as managing booking and capacity in airlines companies [57], health insurances [49,23], scheduling [4], retailers and managers order quantity decision in sports and fashion industries [26].

The basic version of the problem consists in making a one-step decision on the quantity Q to be bought of one single perishable product under the assumption that the demand is a random variable with known distribution F. If the decision maker buys each unit at cost u and sells it at price v, then the expected revenue is maximized by buying exactly $Q^* = F^{-1}(1-\frac{u}{v})$ units.

Numerous variants of the classical SPP have been proposed in the literature; some of them will be discussed below, but for a fuller account of the subject we refer the reader to Khouja [38], Petruzzi and Dada [46], and Qin et al. [47].

The traditional assumption that the demand probability distribution is known may be unrealistic in many cases. In addition, if the demand is inferred from sample data, then the resulting estimate may lack of desirable statistical properties (consistency, asymptotic normality, etc.), for example, for small sample sizes. To overcome these and other related problems, some distribution-free approaches have been considered in the literature, Scarf [50] being the first to give a closed-form solution to the newsvendor problem when only the demand mean and the variance are assumed to be known. Two more remarkable distribution-free works are Gallego and Moon [26], which provided an extension to Scarf's solution, and Yue et al. [58], in which the demand density function is assumed to belong to a specific family of density functions. Other articles which cope with demand uncertainty are Ding et al. [21], Dana and Petruzzi [20], and Godfrey and Powell [28]. However, as pointed out in See and Sim [52] and Bandi and Bertsimas [5], not only the assumption of known distribution of the demand may be too strong, but also to estimate the mean and variance from the sample data and accommodate such estimates to an assumed distribution function may generate drastic errors in the inventory policy. Moreover, demand is in fact usually correlated along time, so assuming that demands for each period are independent and identically distributed is in practice unrealistic [41,29,37]. Some authors have studied inventory models with time-correlated demand, including AR models [2,48,35], compound Poisson processes [53], martingale models of forecast evolution [22,44,56], factor models [52] or estimation via Kalman filter [3]. Most of these papers either assume perfect knowledge of the distribution function [42,2,3,53,56,48] or are focused in calculating bounds of the objective function, which are distribution-

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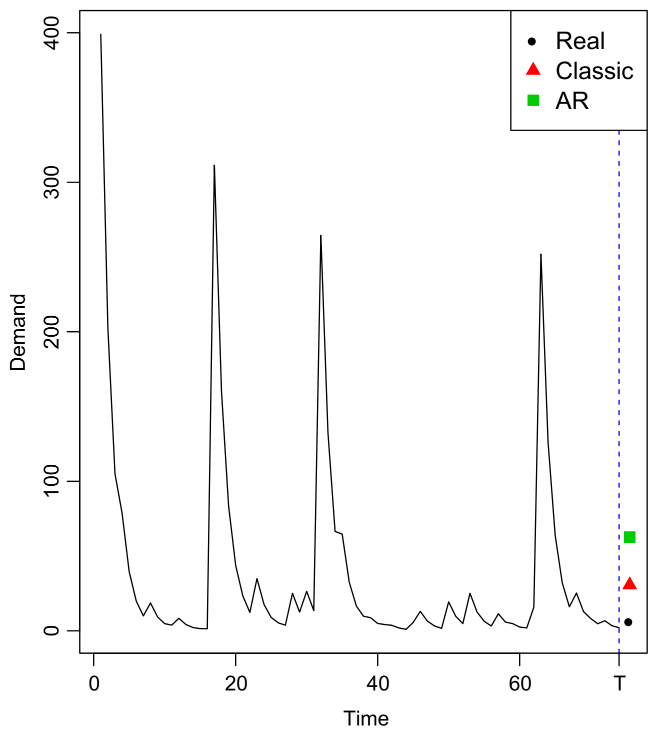

Fig. 1. Demand time series generated from an autoregressive process with heavy-tailed errors, and real and predicted revenues under two classic approaches (classic newsvendor and *AR*).

free in See and Sim [52], Lu et al. [44], and Dong and Lee [22]. In contrast, in the work developed here no distributional assumptions are made and the optimal solution is obtained with a closed expression for a particular case, while the problem to be solved in the remaining cases is extremely tractable due to its structural properties: it is a low-dimensional convex problem. Moreover, we do not only cope with temporal demand but also take into account robustness in terms of uncertainty and risk aversion, which provides novelty to this paper.

Different inventory policies yield rather different revenues. For instance, consider Fig. 1, which depicts a time series for the demand, assumed to follow an autoregressive process with log-normal errors. It can be observed that, in this case, conventional methods such as the classic newsvendor and autoregressive approaches, which will be described in detail in Section 3.1, can lead to losses. In particular, for the data of Fig. 1, the classic model yields a negative revenue of -1.21, while when assuming the basic *AR* model revenues decrease to -9.21. In contrast, robust approaches are usually too conservative and avoid ordering any quantity of product [12,43]. The approach proposed in this paper successfully copes with heavy-tailed demands and usually outperforms both classic and autoregressive approaches in terms of average revenue and also probability of losses, while avoiding over conservativeness of previous robust approaches.

In recent years, risk-averse models have received increasing attention in the inventory literature (see [19] and the references therein). There, instead of maximizing the expected revenues, other utilities are optimized. Depending on the decision maker's preferences, it may be reasonable, for example, optimizing the probability of achieving a target profit (see for example [36] or [39]), the Return of Investment [54], the Cost-Volume-Profit, the CVaR [17] or other risk-averse policies [24,19]. The above-mentioned paper of Yue et al. [58], as well as Perakis and Roels [45], Zhu et al. [60], and Jiang et al. [34], considers the minimax regret decision criterion instead. The robust approach in the newsvendor problem deals with uncertainty in the demand while minimizing the impact over the optimal solution of the worst-case scenario. For example, the landmark Scarf's rule adopts such a criterion, although it enforces independence of the demand along time. Bertsimas and Thiele [12] also propose a robust inventory approach, where uncertainty intervals for the demand are supposed to be already given. On the contrary, our approach would address the worst-case analysis while coping with time-correlated demands and including information of the historical observations of the demand into the model.

In this paper we address the newsvendor problem from a new perspective, integrating a distribution-free design with temporal dependence in the demand, into a robust optimization approach. Throughout the paper we perform a worst-case analysis, seeking the policy maximizing the worst-case revenue. Specifically, our main contributions are:

1. We consider the demand as a time series with non-negligible autocorrelation coefficients. For simplicity, the basic yet versatile autoregressive process of some order p, $AR(p)$, is used as time series model. We follow a distribution-free approach in the sense that no distributional assumption is imposed over the error terms of the autoregressive model.
2. We implement a robust optimization method based on the uncertainty sets of Bandi and Bertsimas [5], where the goal is to minimize the losses in the worst-case realization of the parameters. For the particular case when the uncertainty set is modeled with the l_2-norm, a closed-form expression for the optimal solution is obtained in the case $p=1$. For $p\geq 2$ the problem turns into a tractable nonlinear convex optimization program, solved numerically.
3. We show that our approach outperforms three different classic approaches. In the first one, the demand distribution is assumed to be unknown and is estimated from the sample observations, which are assumed to be independent; in the second one, the demand distribution is assumed to follow an $AR(p)$ process with normal error terms; the third method is the robust distribution-free solution of Scarf [50].
4. We briefly discuss the robust multi-product newsvendor problem with demands correlated over time and between products, and the multi-period case.

The paper is organized as follows. In next section we briefly introduce autoregressive processes and model various robust newsvendor problems with autoregressive demands. Specifically, we formulate the single-item single-period case in terms of an optimization problem in Section 2.1. We discuss the choice of parameters in Section 2.1.1, while in Section 2.1.2 we show that, for a particular case, the problem is a smooth convex optimization problem, and we obtain a closed-form solution for $p=1$. A brief extension to the multi-period case is outlined in Section 2.1.3, where the robust modelling of the autoregressive processes is integrated into the inventory model of Bertsimas and Thiele [12]. An extension to the multiple item newsvendor problem is carried out in Section 2.2, where the demands of the products are assumed to follow a Vector Autoregressive Process. In Section 3 we design and present some numerical examples, where the robust autoregressive model is tested against three different but classic methods, outlined in Section 3.1. Data generation and presentation of results are addressed in Sections 3.2 and 3.3 respectively. Last section is devoted to concluding remarks and extensions.

2. The model

We start this section with a short discussion on autoregressive processes, which will model the demand of our SPP. Because of