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A branch-price-and-cut algorithm for the commodity constrained split delivery vehicle routing problem

Claudia Archetti, Nicola Bianchessi *, M. Grazia Speranza

Department of Economics and Management, University of Brescia, Brescia, Italy

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ABSTRACT

We consider the Commodity constrained Split Delivery Vehicle Routing Problem (C-SDVRP), a routing problem where customers may request multiple commodities. The vehicles can deliver any set of commodities and multiple visits to a customer are allowed only if the customer requests multiple commodities. If the customer is visited more than once, the different vehicles will deliver different sets of commodities. Allowing the splitting of the demand of a customer only for different commodities may be more costly than allowing also the splitting of each individual commodity, but at the same time it is easier to organize and more acceptable to customers. We model the C-SDVRP by means of a set partitioning formulation and present a branch-price-and-cut algorithm. In the pricing phase, the ng-path relaxation of a constrained elementary shortest path problem is solved with a label setting dynamic programming algorithm. Capacity cuts are added in order to strengthen the lower bound. We solve to optimality within 2 h instances with up to 40 customers and 3 commodities per customer.

1. Introduction

The class of vehicle routing problems (VRPs) is one of the largest and most studied classes of combinatorial optimization problems. It is also one of the most computationally challenging classes. In VRPs capacitated vehicles are used to distribute products and satisfy the demand of a set of customers. With very few exceptions, vehicle routing problems model the distribution of a single product. The underlying assumption is that only the volume or the weight of products matters. Thus, even if several products are demanded by customers, the demand of a customer is expressed with a single number, the total weight or volume of the demanded products. Accordingly, the capacity of the vehicles is expressed in weight or volume, depending on which constraint is more binding. We refer to [14] for a recent collection of chapters on different VRPs.

There exist distribution problems where different commodities, or groups of commodities, have to be treated individually. When different commodities require different temperatures, such as in the case of frozen, fresh and dry food, vehicles with compartments may be used. In waste management different kinds of waste cannot be mixed up and must be kept separated at any stage of the collection problem. If a single vehicle is dedicated to the collection, then the vehicle container must be divided in compartments, one for each kind of waste. We refer to [6] for a recent work on a multi-compartment routing problem, with fixed size of the compartments, and to references therein. Also for organizational purposes, different commodities may be handled by means of dedicated vehicles. Having an individual product loaded on a vehicle makes the loading and the unloading of a vehicle much simpler and avoids the need for any reshuffling of the load during the distribution process.

In this paper we consider the Commodity constrained Split Delivery Vehicle Routing Problem (C-SDVRP) introduced in [2], where different commodities are distributed to customers with capacitated vehicles. The vehicles are flexible and can deliver any set of commodities. A customer may be served by more than one vehicle but a single commodity can be delivered to each customer by one vehicle only. Each vehicle starts from a depot, visits a set of customers and returns to the depot at the end of the tour. Any customer may request any set of commodities. A vehicle that carries multiple commodities is totally flexible, that is it can carry any amount of any commodity, provided the constraint on the vehicle capacity is satisfied. We assume that the demand of a commodity of each customer does not exceed the capacity of a vehicle. The problem is a relaxation of the classical Capacitated Vehicle Routing Problem, as multiple visits to customers are allowed. On the other hand, it is more constrained than the Split Delivery Vehicle Routing Problem as the entire demand of a commodity must be delivered by the same vehicle. The concept is that, while allowing split deliveries may be unacceptable to customers, allowing different commodities to be delivered by different vehicles may be

* Corresponding author.
E-mail addresses: claudia.archetti@unibs.it (C. Archetti), nicola.bianchessi@unibs.it (N. Bianchessi), grazia.speranza@unibs.it (M.G. Speranza).

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more acceptable. In addition to the applications mentioned above, the C-SDVRP models the situation where different companies share a warehouse and a fleet of vehicles for the distribution of their products. One commodity is associated with one company. A vehicle may deliver products of different companies to different customers or to the same customers. Different vehicles may serve the same customer only if they deliver products of different companies.

In [2] the C-SDVRP is introduced and compared with alternative ways of distributing multiple commodities, such as by allowing the splitting of individual commodities or by using vehicles dedicated to individual commodities. The tests were performed on 64 small instances, with 15 customers and up to 3 commodities, 80 mid-size instances with 20, 40, 60, 80 customers and up to 3 commodities, and large instances with 100 customers. A branch-and-cut algorithm was able to solve 25 out of 64 small instances to optimality within 30 min. All the remaining instances were heuristically solved by creating multiple copies of each customer, one for each commodity required, and using a heuristic for the Capacitated Vehicle Routing Problem.

In this paper we focus on the exact solution of the C-SDVRP. We formulate the problem through a set partitioning formulation and adopt a branch-price-and-cut approach. In the pricing phase, the ng-path relaxation of a constrained elementary shortest path problem is solved by means of a label setting dynamic programming algorithm. Capacity cuts are added in order to strengthen the lower bound. We tested the algorithm on the small and mid-size instances tested in [2], with a time limit of 2 h. We solved to optimality all small instances within a few seconds, except for three instances for which the computing time is larger than 100 s. For the mid-size instances, we could solve to optimality 19 out of 20 instances with 20 customers and 5 out of 20 instances with 40 customers. For all instances with up to 60 customers, except one, we found a lower and an upper bound. The average and maximum optimality gap is 0.63% and 2.41%, respectively. Over all the mid-size instances, we improved 20 out of 80 best known solutions.

In Section 2 the C-SDVRP is described and in Section 3 the set partitioning formulation is presented. The structure of the branch-price-and-cut algorithm is described in Section 4, together with the formulation and solution of the pricing problem, and the branching scheme. The computational results are presented in Section 5.

2. Problem definition

The Commodity constrained Split Delivery Vehicle Routing Problem (C-SDVRP) can be defined on a directed graph $G=(V,A)$ with vertex set $V=\{0,\dots,n\}$ and arc set $A=\{(i,j): i\neq j; i,j\in V\}$. The vertex set V contains vertex 0, representing the depot, and the set $N=\{1,\dots,n\}$, representing the n customers. A cost c_{ij} is associated with each arc $(i,j)\in A$ and represents the non-negative cost of traversing arc (i,j). Travel costs satisfy the triangle inequality. Let $K=\{1,\dots,m\}$ be the set of commodities that have to be distributed from the depot to the customers. The demand of commodity $k\in K$ to be delivered to customer $i\in N$ is denoted by d_{ik}. The set $K_i=\{k\in K \mid d_{ik}>0\}$ contains the commodities to be delivered to customer $i\in N$. We define as F the fleet of identical vehicles that is available to serve the customers and as Q the vehicle capacity. Vehicles are flexible and can deliver any subset of commodities. Each customer may be visited more than once. When a commodity is delivered by a vehicle to a customer, the entire amount requested by the customer is provided. Thus, multiple visits to a customer are allowed only if the customer requests multiple commodities. If a customer receives multiple visits, it means that the different vehicles will deliver different commodities. The objective is to find a set of routes serving all the customers in such a way that the total traveling cost is minimized.

3. Formulation

We model the problem by means of a set partitioning formulation making use of an exponential number of variables, each associated with a different feasible route. We adopt the following notation. We define as R the set of all feasible routes. A route corresponds to a non-empty cycle in graph G starting from and ending at the depot. For each route $r\in R$, let $c^r=\sum_{(i,j)\in r}c_{ij}$ be the cost associated with the route. Then, let a^r_{ik}, e^r_i and b^r_{ij} be binary coefficients equal to 1 if commodity k is delivered to customer $i\in N$, customer $i\in N$ is visited, and arc $(i,j)\in A$ is traversed in route r, respectively, and 0 otherwise. The C-SDVRP can be formulated as follows:

$$\min \sum_{r\in R} c^r\lambda^r \tag{1}$$

$$\text{s.t.}: \quad \sum_{r\in R} a^r_{ik}\lambda^r \geq 1, \quad i\in N,\ k\in K_i \tag{2}$$

$$\sum_{r\in R}\lambda^r=\phi \tag{3}$$

$$\sum_{r\in R} e^r_i\lambda^r = z_i, \quad i\in N \tag{4}$$

$$\sum_{r\in R} b^r_{ij}\lambda^r = x_{ij}, \quad (i,j)\in A \tag{5}$$

$$1\leq z_i\leq \max\{|K_i|,|F|\} \text{ and integer}, \quad i\in N \tag{6}$$

$$\left\lceil \frac{\sum_{i\in N}\sum_{k\in K_i} d_{ik}}{Q}\right\rceil \leq \phi \leq |F| \text{ and integer} \tag{7}$$

$$0\leq x_{ij}\leq |F| \text{ and integer}, \quad (i,j)\in A \tag{8}$$

$$\lambda^r\in\{0,1\}, \quad r\in R, \tag{9}$$

where x_{ij} and z_i are integer variables representing the number of times the vehicles traverse arc $(i,j)\in A$ and visit customer $i\in N$, respectively, ϕ is an integer variable representing the number of vehicles used, and λ^r is a binary variable equal to 1 if route $r\in R$ is assigned to a vehicle. The objective function (1) aims at minimizing the total traveling cost. Constraints (2) ensure that all commodities requested by each customer will be delivered. Constraints (3) and (7) bound the number of vehicles that can be used. Constraints (4) and (6) bound the number of times a customer $i\in N$ can be visited. Constraints (5) and (8) bound the number of times an arc $(i,j)\in A$ can be traversed. Finally, (9) state that λ^r are binary variables. Note that x_{ij} can take any integer value between 0 and $|F|$ contrary to what happens in the Split Delivery Vehicle Routing Problem (SDVRP) where x_{ij} is a binary variable for $i,j\in N$. In fact, in [2] the authors show that in the C-SDVRP arcs joining two customers can be traversed more than once.

4. Branch-price-and-cut algorithm

In the following we describe the algorithm designed to solve the set partitioning formulation (1)–(9) which will be referred to as the Master Problem (MP).

In order to solve the MP we design a branch-price-and-cut algorithm [4,7], that is a branch-and-bound algorithm where, at each node of the branch-and-bound tree, λ^r variables are generated by means of column generation while addressing the linear