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A leader–follower model for discrete competitive facility location

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ABSTRACT

In this paper we investigate a leader–follower (Stackelberg equilibrium) competitive location model. The competitive model is based on the concept of cover. Each facility attracts consumers within a "sphere of influence" defined by a "radius of influence." The leader and the follower, each has a budget to be spent on the expansion of their chains either by improving their existing facilities or constructing new ones. We find the best strategy for the leader assuming that the follower, knowing the action taken by the leader, will react by investing his budget to maximize his market share. The objective of the leader is to maximize his market share following the follower's reaction.

1. Introduction

There is a rich body of literature dealing with competitive location models. Such models are applicable to the location of competing facilities, such as retail stores, shopping centers, restaurants and others. These competing facilities may be chain facilities and franchises. Some of them belong to one's own chain while others are referred to as the competitor. Competing facilities that provide similar products exist in the market and compete for consumers' patronage. The problem is to find the best location for one or more new facilities in a market where competition already exists or will exist in the future. The objective is to locate the retail outlet at the location that maximizes its market share. Revenue increases when market share increases. Thus, maximizing profit is equivalent to maximizing market share when construction and operating costs are about the same at each location. Dasci and Laporte [9], Jensen [36], Winerfert [52] discuss this issue. For a review of competitive location models the reader is referred to [7,13,22,23,26].

The underlying theme of competitive models is the existence of an interrelationship among four variables: buying power (demand), distance, facility attractiveness, and market share, with the first three variables being independent variables and the market share the dependent variable. Many rules were proposed for formulating this relationship. Hotelling [33] analyzed a situation on a line with distance patronizing behavior from a game theoretic point of view. He assumed that each consumer patronizes the closest facility. Eiselt [24] and Eiselt and Laporte [25] extended the game theoretic approach to a tree environment. The proximity rule was generalized to a utility function or random utility rule by [10,14,38]. Huff [34,35] introduced a patronizing model and investigated its validity with data. He suggested that consumers distribute their patronage according to a probabilistic gravity model suggested by Reilly [44]. Drezner et al. [18,19] proposed a cover-based rule. Each facility has a catchment area attracting customers residing within it.

Many extensions to the basic model were suggested. The following two extensions are incorporated in this paper.

Budget constraints: Combining the location decision with facility design (treating the attractiveness level of the facility as a variable) was recently investigated in [1,11,18,19,27,39,42,50]. Drezner [11] assumed that the facilities' attractiveness are variables. In that paper it is assumed that a budget is available for locating new facilities and for establishing their attractiveness levels. One needs to determine the facilities' attractiveness levels so that the available budget is not exceeded. Plastria and Vanhaverbeke [40] combined the limited budget model with the leader–follower model. Aboolian et al. [1] studied the problem of simultaneously finding the number of facilities, their respective locations and attractiveness (design) levels.

Leader–follower: The leader–follower model (Stackelberg [47]) considers a competitor's reaction to the leader's action. The leader decides to expand his chain. The follower is aware of the action taken by the leader and expands his facilities to maximize his own market share. The leader's objective becomes maximizing his market share *following* the follower's reaction. The leader–follower location

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model in a competitive environment was investigated in [15,20,37,40–42,45,46].

In this paper we analyze and solve the leader–follower model incorporating facilities' attractiveness (design) subject to limited budgets for both the leader and follower. We investigate what is the main source of extra market share for the leader and the follower. Is it attracting new consumers that did not patronize any facility prior to the expansion, or is it attracting competitor's customers.

The paper is organized as follows: In Section 2, the estimation of market share applied in this paper is discussed and the model is formulated in Section 3. In Section 4, the tabu search solution procedure for the leader's problem is detailed. In Section 5, the results of extensive computational experiments are reported followed by conclusions in Section 6.

2. Estimating market share

In this paper we apply the cover-based competitive location model [18,19] for estimating market share. Each facility has a sphere of influence (catchment area) for patronage defined by a distance termed radius of influence. Consumers patronize a facility if they are located within the facility's radius of influence. More attractive facilities have a larger radius of influence, thus they attract consumers from greater distances. Demand at demand points which are not attracted by any facility is lost.[2] When the total captured market share is estimated, we assume that if a consumer is attracted to more than one facility, his/her buying power is equally divided between the attracting facilities. A comprehensive discussion of this rule is presented in Section 2 of [18]. Equal division may not be accurate for a single consumer but the aggregated market share is estimated reasonably well as pointed out in [18].

This rule is much simpler to implement than gravity models or utility-based models. We only need to estimate the catchment area of competing facilities which yields their radius of influence. There are established methods for estimating the radius of influence of a facility [5,49]. For example, license plates of cars in the parking lot are recorded and the addresses of the cars' owners obtained. Drezner [12] conducted interviews with consumers patronizing different shopping malls asking them to provide the zip code of their residence and whether they came from home.

When the gravity rule [34,35] is applied, one needs to determine the distance decay function and then determine the parameters of this function. The distance decay function $f(d)$ represents the decline in facility attractiveness as a function of the distance from the facility and thus the probability that a consumer patronizes a facility. In the original gravity model [44], it is assumed that the distance decay parallels gravity decay and thus $f(d)=1/d^2$. Huff [34,35] suggested a decay function of $f(d)=1/d^{\lambda}$ where the power λ depends on retail category. $\lambda=3$ was found for grocery stores [35], $\lambda=3.191$ for clothing stores [34], $\lambda=2.723$ for furniture stores [34], and $\lambda=1.27$ for shopping malls [12]. Wilson [51] suggested an exponential decay $e^{-\lambda d}$ which was used in many subsequent papers [1–3,16,32]. Drezner [12] compared power and exponential decay on a real data set of shopping malls in Orange County, California and found that exponential decay fits the data better than power decay. The decay function $f(d)=e^{-1.705d^{0.409}}$ was used in [6] who investigated grocery stores. A Logit function $f(d)=1/(1+e^{\alpha+\beta d+\gamma d^2})$ was used in [21].

Different approaches may yield different estimates of the market share. It is not clear which decay function should be used and what value of λ should be applied. It is also not clear that estimating the market share by complicated models (and assigning a parameter λ which may not be accurate) is more accurate than using the cover-based simple approach. Note that in the cover-based approach some of the demand is lost because it is not attracted to any facility. Most other models assume that all demand are satisfied, which is rarely a reasonable assumption. It should also be noted that the majority of competitive location papers, which apply a gravity model for estimating market share, assume that λ is "known" and the authors perform computational experiments with its arbitrarily selected value to illustrate their algorithms.

The same is true for other approaches for estimating market share which require numerous parameters for their implementation. Our approach requires only the establishment of the catchment area. All models that assume a budget constraint in their formulation must determine the cost of establishing and improving facilities of a given attractiveness.

3. Formulation

We employ the cover-based model [18,19]. In Drezner et al. [18], the location of p new facilities with a given radius is sought so as to maximize the market share captured by one's chain. In Drezner et al. [19], three strategies were investigated: In the improvement strategy (IMP) only the improvement of existing chain facilities is considered; in the construction strategy (NEW) only the construction of new facilities is considered; and in the joint strategy (JNT) both improvement of existing chain facilities and construction of new facilities are considered. All three strategies are treated in a unified model by assigning a radius of zero to potential locations of new facilities.

The leader employs one of the three strategies and the follower also implements one of these three strategies. This setting gives rise to nine possible models. Each model is a combination of the strategy employed by the leader and the strategy employed by the follower. For example, the leader employs the JNT model, i.e. considers both improving existing facilities and establishing new ones, while the follower may employ the IMP model, i.e. only considers the improvement of his existing facilities. The most logical model is to employ for both the leader and the follower the JNT strategy which yields the highest market share. However, constructing new facilities or improving existing ones may not be a feasible option for the leader or the follower.

3.1. Notation

The set of potential locations for the facilities is discrete.

1. N, the set of demand points of cardinality n.
2. w_i, the buying power at demand point i, $i=1,\dots,n$.
3. L_i, the number of facilities that belong to the leader's chain that attract demand point i.
4. F_i, the number of follower's facilities attracting demand point i.
5. B_L, the budget available to the leader for increasing the attractiveness of existing facilities or constructing new ones.
6. B_F, the budget available to the follower for increasing the attractiveness of existing facilities or constructing new ones.
7. P_L, the set of the existing leader's facilities including potential locations for new facilities of cardinality p_L.

[2] It should be noted that most models assume that all the buying power is distributed among the competing facilities. Lost demand is addressed in [2,8,16,17,43].