Computers & Operations Research 64 (2015) 189–197

Contents lists available at ScienceDirect

Computers & Operations Research

journal homepage: www.elsevier.com/locate/caor

Review

Time-dependent routing problems: A review

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ARTICLE INFO

Available online 14 June 2015

Keywords:
Travel time models
Quickest path problem
Vehicle routing problem

ABSTRACT

Time-dependent routing amounts to design "best" routes in a graph in which arc traversal times may vary over the planning horizon. In the last decade, a number of technological advances have stimulated an increased interest in this field. We survey the research in the area and present a comprehensive review of travel time modelling, applications and solution methods. In particular, we make a first classification in point-to-point and multiple-point problems. A second major classification is then performed with respect to the quality and evolution of information. Other criteria included: (i) node, arc or general routing; (ii) the possibility to choose the vehicle speed.

Contents

1. Introduction

Routing is the process of selecting "best" routes in a graph $G=(V,A)$, where V is a node set and A is an arc set. Majority of the studies on routing problems have been conducted under the assumption that all the information necessary to formulate the problems is time-invariant [88]. In practical applications, this assumption is usually not verified given the presence of time varying traversal times. Travel times may vary *exogenously* due to traffic congestion, weather conditions, moving targets or mobile obstacles, or *endogenously* whenever the decision maker can set the vehicles' speeds (e.g., in order to trade-off between fuel consumption and travel time).

With each arc $(i,j) \in A$ is associated a function $\tau_{ij}(t)$ representing the time a vehicle, leaving at instant t, takes to traverse the arc. Given a start time t and a route $p=(s=v_1,v_2,\ldots,v_k=d))$ from a

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http://dx.doi.org/10.1016/j.cor.2015.06.001
0305-0548/© 2015 Published by Elsevier Ltd.

node $s \in V$ to a node $d \in V$ (possibly coincident with s), its *traversal time* $z_p(t)$ is defined recursively as

$$z_{(v_1,\ldots,v_i)}(t) = z_{(v_1,\ldots,v_{i-1})}(t) + \tau_{(v_{i-1},v_i)}(t + z_{(v_1,\ldots,v_{i-1})}(t)), \tag{1}$$

with the initialization $z_{(v_1,v_2)}(t) = \tau_{(v_1,v_2)}(t)$.

Time-dependent routing problems arise naturally in a variety of applications, including route planning in road networks, travel planning in public transit networks, vehicle routing problems (in particular, vessel routing) as well as some robotic and military applications. We now review the main application areas.

Route planning in road networks, i.e., the calculation of the most effective route from an origin to a destination node on a road network, is a key component of both car navigators and web-based *travel information services*. Although at the time of writing live and historic data are available in more than 600 areas in over 50 countries, time-dependent route planning is not a service provided yet (see, e.g., [38]). One of the reasons is that the state-of-the-art algorithms for the time-dependent quickest path problem are still relatively slow (they take milliseconds on continental-sized networks while their time-invariant counterparts require microseconds). This is a major issue in a typical web application setting where a server needs to answer several quickest path queries in a fraction of a second on graphs with millions of nodes [21].

Travel planning in public transit networks, i.e., finding the quickest connection in a transportation network (e.g., a bus, train, ferry or multimodal network) is inherently event-based and hence time-dependent in nature [39].

Time dependent models also play a major role in a variety of *vehicle routing problems* (VRPs) arising in distribution planning, mail delivery, garbage collection, salt gritting, field service routing, etc. Data on current traffic conditions can be used to enhance fleet management in two ways: by taking into account explicitly the historical traffic patterns while building a priori routes; by re-routing vehicles in real-time (if operationally feasible) as soon as updated traffic information become available. Eglese et al. [24] examine the issues involved in constructing a database of time-dependent traversal times for a road network and assess the benefits of time-dependent vehicle routing and scheduling systems in a distribution application in the Northwest of England. A peculiar class of time-dependent VRPs arises when planning the route of an aircraft, a ship or a submarine in a two or three-dimensional space. Here the decisions include not only the routing part but also the power setting. The objective may include the minimization of travel time, fuel consumption or a combination of them [72,69]. The problem has received attention in recent years thanks to the recent advancements in sensor and data-processing technology that facilitate the collection of detailed real-time information about the air currents, the surface of the ocean or the underwater flow velocity. Another source of time-dependency may be caused by the need to resupply moving units, like patrolling boats [46], or when an aircraft must intercept a number of mobile ground units [51]. Finally, time dependent routing problems arise in *robot motion planning* in the presence of moving obstacles [82,55,33].

Time-dependent routing problems may be classified with respect to a number of criteria. In our review, we have chosen to make a first classification between point-to-point and multiple-point problems. A second major classification is performed with respect to the quality and evolution of information as in [74] (see Table 1). Other criteria include: (i) node, arc or general routing depending on whether the customers are associated to nodes, arcs or both; (ii) the possibility to choose the vehicle speed.

2. Travel time and speed models

Travel time functions can be usually assumed to be continuous in road networks. On the other hand, they are discontinuous in nature in transit networks. As shown in Fig. 1, the presence of a discrete number of trips between a node $i \in A$ and $j \in A$ makes the travel time $\tau_{ij}(t)$ equal to the sum of a constant trip duration and a waiting time at node i (the difference between the start time of the next trip and t). It is worth noting that in a transit network it may be convenient to wait at a node: for instance, a traveler standing on a platform in a railway station may prefer not to board on the next local train and to wait for the subsequent express train to his destination. Routing problems on transit networks are often transformed into time-invariant problems on time-expanded graphs [9] in which a vertex is created for each event in the timetable (e.g., a vehicle departure or arrival at a stop) and an arc is inserted for every connection between its respective departure and arrival events or with the aim of linking subsequent connections. Such arcs are then weighted by the time difference between

Table 1
Taxonomy based on the quality and evolution of information proposed by [74].

Information evolution	Information quality	
	Deterministic input	**Stochastic input**
Input known beforehand	Static and deterministic	Static and stochastic
Input changes over time	Dynamic and deterministic	Dynamic and stochastic

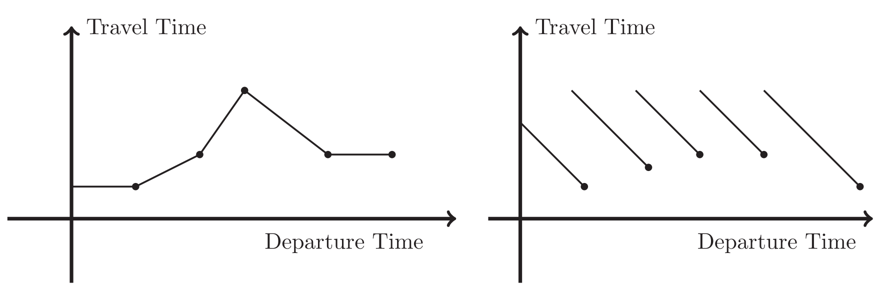

Fig. 1. Example of piecewise linear travel time, the left figure refers to road networks while the right one deals with railway networks.