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An action-space-based global optimization algorithm for packing circles into a square container

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ABSTRACT

This paper proposes an action-space-based global optimization (ASGO) approach for the problem of packing unequal circles into a square container such that the size of the square is minimized. Starting from several random configurations, ASGO runs the following potential descent method and basin-hopping strategy iteratively. It finds configurations with the local minimum potential energy by the limited-memory BFGS (LBFGS) algorithm, then selects the circular items having the most deformations and moves them to some large vacant space or randomly chosen vacant space. By adapting the action space defined for the rectangular packing problem, we approximate each circular item as a rectangular item, thus making it much easier to find comparatively larger vacant spaces for any given configuration. The tabu strategy is used to prevent cycling and enhance the diversification during the search procedure. Several other strategies, such as swapping two similar circles or swapping two circles in different quadrants in the container, are combined to increase the diversity of the configurations. We compare the performance of ASGO on 68 benchmark instances at the Packomania website with the state-of-the-art results. ASGO obtains configurations with smaller square containers on 63 instances; at the same time it matches or approaches the current best results on the other five instances.

1. Introduction

The packing problem is a well-known NP-hard problem, and it is concerned with how to pack a certain number of circles of given radius inside a container with no overlap. The shape of the container can be a circle, a rectangle, a square or a polygon, and the items can be circles, rectangles or irregular items. The packing problem has a wide range of applications in the areas of marine transportation, motor cycle industry, material cutting, fashion industry, wireless communication, food industries, etc. As an NP-hard problem, there exists no exact algorithm for solving it to optimality in polynomial time unless P=NP.

If the items to be packed are circles, the problem is called the circles packing problem (CPP), which has been subject of study by a wide spectrum of different approaches in the literature. The problem can be classified into two categories, in accordance with the items being equal circles or unequal circles. Besides, there is an important extension of the circle packing problem with equilibrium constraints (CPPEC) [1,2].

For the problem of packing equal circles, most heuristic approaches are based on a quasi-physical or quasi-human method. Lubachevsky and Graham [3] proposed a billiards simulation algorithm based on the collision forces among objects: they regarded each cycle item as a rigid billiard and considered their movements under the collision force. They also proved that their algorithm could obtain optimal solutions when the number of circles n equals $3k(k+1)+1$ for any positive integer k. Grosso et al. [4] presented a genetic algorithm based on the monotonic basin-hopping (MBH) strategy and on a population basin-hopping strategy. Huang and Ye [5,6] regarded each item as an elastic circle. They also considered the smooth movement driven by elastic forces and the violent movement driven by strong repulsive forces and attractive forces and proposed a quasi-physical global optimization method. By adapting an improved energy-landscape-paving method (ELP), Liu et al. [7] incorporated a new configuration update mechanism into the ELP method.

For the problem of packing unequal circles into a larger container, the methods can be classified into two categories: constructive heuristics and global search heuristics. For the constructive heuristics, George et al. [8] formulated this situation as a nonlinear mixed integer programming problem and developed some heuristic procedures, including a genetic algorithm and a quasi-random technique. Huang et al. [9] defined the concept of hole degree in order to select the next circle to place and applied a lookahead search strategy. Lü and Huang

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[10] incorporated the pruned-enriched Rosenbluth method (PERM) into the strategy of the maximum cave degree. The PERM strategy was to prune and enrich branches efficiently, while the concept of maximum hole degree was defined to evaluate the benefit of a partial configuration. By improving the algorithms B1.0 and B1.5 [11], Kubach et al. [12] formulated some greedy algorithms, and they parallelized these algorithms by utilizing a master-slave approach followed by a subtree-distribution model. Akeb and Hifi [13] proposed an augmented algorithm which combined a beam search, a binary search, and the multi-start strategy; they also incorporated a strategy based on separate beams, instead of pooled ones, to increase the efficiency of the algorithm. For the global search heuristics, Stoyan and Yas'kov [14] utilized the reduced gradient method for local optimization, then translated one local minimum to another local minimum based on the concept of active inequalities and the Newton method. Hifi et al. [15] defined an energy function for the local optimization, and by utilizing some configuration transformation methods, they proposed a simulated annealing approach. Fu et al. [16] proposed an iterated tabu search procedure to improve the randomly generated solution, then a perturbation operator was subsequently employed to reconstruct the current solution and an acceptance criterion was implemented to determine whether to accept the perturbed solution or not. Lopez and Beasley [17] viewed the problem as being one of scaling the radii of the unequal circles so that they could be packed into the container, and their algorithm was also composed of an optimization phase which was based on the formulation space search method, while the improvement phase created a perturbation of the current solution by swapping two circles.

In 2011 and 2012, He et al. proposed the concept of action space for solving the rectangular packing problem. An action space in a configuration is an unoccupied rectangular space that a dummy rectangle could be feasibly placed in and each edge of the dummy rectangle pastes at least one placed item or the boundary of the container. Inspired by their concept of action space, we propose an action-space-based global optimization (ASGO) algorithm for the problem of packing unequal circles into a square container (PUCS). There are three procedures in ASGO. Given some configurations, we utilize the LBFGS [18] algorithm for continuous optimization to reach the local minimums. Then we approximate each circular item as a rectangular item and an action space based basin-hopping strategy is adapted to find the larger vacant spaces such that the search procedure could jump from a local minimum to a more promising area. A post processing is added to improve the accuracy of the result. In the experiments, numerical results are presented and compared with the best-known results taken from the Packomania website maintained by Specht [19].

2. Problem formulation

In the problem of packing unequal circles into a square container (PUCS), we are given $n (n \in N^+)$ circle items $C_1, \ldots, C_n$, each circle having radius r_i, and we want to find a nonoverlapping dense packing of the circle items into a square container such that the size W of the square container is as small as possible. If we place the center of the square at the origin of a 2D Cartesian coordinate system, and denote the center coordinate of item C_i as (x_i, y_i)(as shown in Fig. 1(a)), then $X = (x_1, y_1, x_2, y_2, \ldots, x_n, y_n)$ could uniquely denote a packing configuration. Define $D_{ij} = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2}$, which indicates the Euclidean distance between the center of C_i and C_j, as shown in Fig. 1(b). The problem can be formulated as follows:

$$\begin{aligned} &\min \quad W \\ &\text{s.t.} \quad \text{(i)}\ (|x_i| + r_i, |y_i| + r_i)^+ \le 0.5W \\ &\qquad\ \ \text{(ii)}\ D_{ij} \ge r_i + r_j \end{aligned}$$

where the symbol $(x, y)^+ \triangleq \max(x, y)$ for variables x and y, and i, j imply to $1, 2, \ldots, n; i \ne j$. Constraint (i) indicates that each circle should be placed completely in the container. Constraint (ii) indicates that any pair-wise circles must not overlap each other.

Regard all the circles as smooth elastic disks and regard the square container as a rigid hollow object. According to the elastic mechanics, the elastic deformation, caused by the overlaps between two disks or between a disk and the borders of the container, will generate elastic potential energy, which can be used to measure the feasibility of a given configuration (X, W). We define the overlapping depth of C_i and C_j as follows:

$$d_{ij} = (r_i + r_j - D_{ij})^+ \tag{1}$$

where symbol $x^+ \triangleq \max(x, 0)$ for a variable x. The overlapping depths between the disk C_i and respectively the vertical or horizontal

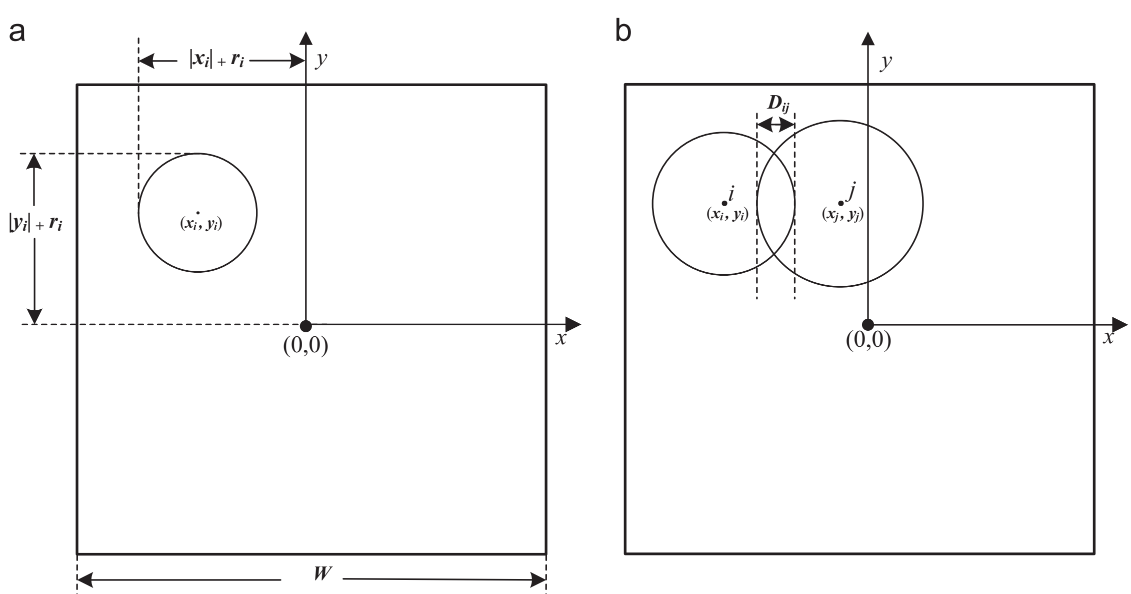

Fig. 1. Constraints of the PUCS problem.