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A space-discretized mixed-integer linear model for air-conflict resolution with speed and heading maneuvers

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ABSTRACT

Air-conflict resolution is a bottleneck of air traffic management that will soon require powerful decision-aid systems to avoid the proliferation of delays. Since reactivity is critical for this application, we develop a mixed-integer linear model based on space discretization so that complex situations can be solved in near real-time. The discretization allows us to model the problem with finite and potentially small sets of variables and constraints by focusing on important points of the planned trajectories, including the points where trajectories intersect. A major goal of this work is to use space discretization while allowing velocity and heading maneuvers. Realistic trajectories are also ensured by considering speed vectors that are continuous with respect to time, and limits on the velocity, acceleration, and yaw rate. A classical indicator of economic efficiency is then optimized by minimizing a weighted sum of fuel consumption and delay. The experimental tests confirm that the model can solve complex situations within a few seconds without incurring more than a few kilograms of extra fuel consumption per aircraft.

1. Introduction

1.1. Automating air traffic control

Travelers would certainly hesitate before taking a flight if they thought there was a possibility that their aircraft would collide with another. It is the mission of air traffic control (ATC) operators to ensure in real-time that this fear remains unfounded. Based on reference horizontal and vertical separation distances, the event that must be avoided is a *loss of separation*, i.e., a simultaneous violation of both reference distances. When a loss of separation is predicted between two aircraft, they are said to be involved in a *conflict*. The ATC operators thus monitor the traffic, and when they detect a conflict, they design maneuvers that avoid a loss of separation. These maneuvers may involve a modification of speed, heading, or altitude, although altitude changes are rare for several reasons including the comfort of passengers, fuel consumption, and ease of monitoring.

Monitoring the traffic and designing maneuvers generates a heavy cognitive workload for the ATC operators. This would compromise safety if anticipative actions were not carried out to avoid dangerous situations. ATC is thus a critical stage in the overall air traffic management (ATM), since it constrains the airspace structure and traffic flow. Assuming that the forecast published by Eurocontrol [25] is correct, air traffic will undergo a 50% increase by 2035. In this context, severe additional regulations will be needed to deliver reasonable aircraft flows to ATC. A simulation-based study by Lehouillier et al. [13] estimates that this will lead to a 34-fold growth in delay costs if the current control procedures are unchanged. It seems that automated tools for ATC will be necessary to support this important traffic increase. Since the conflict resolution problem is the most complex task of ATC operators and presents a mathematical and algorithmic challenge, it has been widely studied.

1.2. Literature review on automated conflict resolution

The conflict resolution problem consists in finding minimum-cost trajectories connecting two given positions with the linking constraints that a minimum separation should be ensured at all times. The result is a difficult continuous-time problem in which multiple trajectories must be simultaneously determined with additional nonconvex separation constraints. As a consequence, the existing studies all represent a compromise between the generality of the hypotheses, the realism of the model, and computational efficiency.

The most realistic models are developed in the theoretical framework of optimal control [5,20]. The continuous-time nature of the problem is conserved, but analytical solutions can be found only for simple cases with two aircraft and a constant velocity. Based on the shapes of these analytical solutions, Bicchi and Pallottino [5] handle

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situations involving more aircraft and conflicts with a heuristic approach. The algorithm minimizes the total flown distance, but limits the exploration to constant speed trajectories combining circular arcs and straight lines. Raghunathan et al. [20] sample the time interval to develop a nonlinear program (NLP) that may be solved numerically. However, Borrelli et al. [6] observed rather disappointing computational results when solving the NLP, highlighting the importance of the starting point.

Several alternatives have been proposed for a rapid solution. Frese and Beyerer [11] limit the possible maneuvers to a finite set of instantaneous heading changes with constant speed or instantaneous speed changes with constant heading to solve the problem through a tree exploration. Focusing also on a finite set of maneuvers involving heading changes with constant speed, several authors have applied metaheuristic methods. For instance, Durand and Alliot [9] implement an ant colony algorithm, and Alonso-Ayuso et al. [3] develop a variable neighborhood search.

The most widespread family of algorithms is based on mixed-integer linear programs (MILPs). Moving from a complete continuous-time formulation to an MILP requires a discretization of the optimal control problem so that it can be modeled with a finite number of constraints and variables. The most obvious discretization is that used to obtain an NLP in [20]: the time interval is sampled into a finite set of time steps, and decisions are taken at each step. The separation constraints are then verified by measuring the distance between each pair of aircraft at each time step. An MILP with *time discretization* is for instance developed by Schouwenaars [23], by Richards and How [22] and by Omer and Farges [17]. Another strategy is to resolve the conflicts with at most one maneuver per aircraft, executing all the maneuvers simultaneously at the initial time. This may be interpreted as a particular case of time discretization in which only one time step is considered; see [19,26].

Since the entire space may not be relevant for the conflict resolution, another discretization method, henceforth referred to as *space discretization*, focuses on the most interesting points of the airspace, namely those where trajectories intersect. In contrast to time discretization, the natural decision variables then represent the instants when the sampling points are flown over. Separation is then characterized by the gap between fly over times at conflict points. This technique is used to develop models that are restricted to speed maneuvers in [27,2,21].

This classification of discretization techniques may also be extended to several nonlinear models. For instance, a time discretization is used in [4] and a space discretization is used in [7] to solve conflicts with speed changes only.

1.3. Critical analysis and contribution statement

The strength of MILPs is that algorithms can guarantee to find optimal solutions, and efficient implementations of these algorithms are fast even for large numbers of variables and constraints. For this particular application, optimality may seem insignificant since reasonable maneuvers impact only small portions of the complete trajectories. Good conflict-free trajectories, such as those determined by controllers, should not cost much more than the optimal ones. However, MILP approaches guarantee that if a feasible solution exists it will be found, and it will provide an efficient way to resolve the conflicts as long as the objective function accurately reflects real costs.

Of the modeling techniques that have been used to formulate the problem with linear constraints and objective function, time discretization appears to be classical. It enables several authors to include both speed and heading maneuvers while taking realistic constraints into account [20,6,10,17]. The disadvantage of these models is that they have to sample the time horizon with a sufficiently large number of time steps to remain precise. This leads to an equally large number of variables and constraints and a potentially large computational time. In contrast, spatial discretization focuses on a small number of interesting points, so the problem is expected to be solved quickly. However, the sampling points directly depend on the predicted trajectories, which makes it hard to represent the geographical deviations that would result from heading maneuvers. For this reason, models based on a spatial discretization allow only speed maneuvers [21,7] or also include instantaneous altitude changes reflecting a flight-level assignment rather than dynamic altitude maneuvers [27,2]. The drawback of this assumption is that aircraft cannot dramatically modify their speed at will for reasons such as aircraft performance, passenger comfort, fuel consumption, or delay adjustments. Altitude maneuvers are usually performed to separate aircraft with a vertical motion or as last-resort safety measures.[1] In the context of this paper, the heading maneuvers correspond better to the preferences of controllers and pilots.

Moreover, since the existing models implicitly assume small speed changes that comfortably anticipate the conflicts, they leave aside several realistic features that appear in some time-discretized formulations. For instance, these space-discretized models minimize the remaining losses of separation or the amplitudes of speed changes although airlines are mostly interested in fuel consumption and delays. They also include instantaneous speed changes although acceleration should be limited.

Our main contribution is to develop a space-discretized model that allows both speed and heading maneuvers. We attempt to make the designed trajectories more realistic and to comply with operational needs arising from the traffic flow management or from the airlines.

As a consequence, this study considers that

- speed vectors are continuous functions of time;
- acceleration vectors are bounded to respect the comfort of passengers;
- maneuvers should minimize the total fuel consumption;
- aircraft should revert to their planned trajectories and minimize delay.

The complexity of a situation depends on the number of aircraft it involves, on the number and on the interdependency of the conflicts, and on the geometric structure. To evaluate the model, we generate a large benchmark of artificial instances involving up to 12 aircraft engaged in 36 simultaneous potential conflicts. A recently developed time-discretized model is used as a reference for experimental comparisons. The experiments aim to provide a proof of concept for the space-discretized model and to assess its ability to find efficient and realistic trajectories in a few seconds while ensuring separation for complex situations.

Our approach is based on the problem definition and on the principles of space discretization presented in Section 2. To formulate the problem with linear constraints, the maneuvers need to be restricted to particular patterns that are described and studied in Section 3. The overall model resulting from these assumptions is then developed in Section 4. It is evaluated and analyzed through experimental tests on a large number of data sets in Section 5.

2. Discretizing the problem spatially

2.1. Problem definition

The conflict resolution problem aims to keep a set of aircraft $\mathcal{A}$ separated on a time interval $[0, T]$. Let $\mathcal{C}$ be the set of pairs of

[1] See the Eurocontro webpage http://www.eurocontrol.int/acas.