Computers & Operations Research 43 (2014) 20–35

Contents lists available at ScienceDirect

Computers & Operations Research

journal homepage: www.elsevier.com/locate/caor

The asymmetric bottleneck traveling salesman problem: Algorithms, complexity and empirical analysis ☆

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ARTICLE INFO

Available online 19 August 2013

Keywords:
Traveling salesman problem
Bottleneck problems
Experimental analysis
Heuristics
Complexity
Approximation algorithms

ABSTRACT

We consider the asymmetric bottleneck traveling salesman problem on a complete directed graph on n nodes. Various lower bound algorithms are proposed and the relative strengths of each of these bounds are examined using theoretical and experimental analysis. A polynomial time $\lceil n/2 \rceil$−approximation algorithm is presented when the edge-weights satisfy the triangle inequality. We also present a very efficient heuristic algorithm that produced provably optimal solutions for 270 out of 331 benchmark test instances. Our algorithms are applicable to the maxmin version of the problem, known as the maximum scatter TSP. Extensive experimental results on these instances are also given.

1. Introduction

Let $G=(V,E)$ be a directed or undirected graph with $n=|V|$ and $m=|E|$. For each edge $(i,j) \in E$, a nonnegative cost c_{ij} is prescribed. Without loss of generality, we assume that G is complete. The $n \times n$ matrix $C=(c_{ij})_{n \times n}$ is called the cost matrix. Let $\Pi(G)$ be the collection of all (directed) Hamiltonian cycles in G. Then the bottleneck traveling salesman problem (BTSP) [20] is to find a Hamiltonian cycle (tour) in G whose largest edge cost is as small as possible, i.e.

$$\text{Minimize} \quad \max\{c_{ij} : (i,j) \in H\}$$
$$\text{subject to} \quad H \in \Pi(G). \tag{1}$$

Akin to the traveling salesman problem (TSP), BTSP instances are classified as either symmetric (i.e. $c_{ij}=c_{ji}$ for all $i,j \in V$) or asymmetric (i.e. $c_{ij} \neq c_{ji}$ for some $i,j \in V$).

BTSP is a special case of the minmax combinatorial optimization problem [37]. For a complete discussion on the complexity of the BTSP we refer to the book chapter by Kabadi and Punnen [26]. In particular, the BTSP is NP-hard, and, unless P=NP, no polynomial time ϵ−approximation algorithm exists for the problem for any $\epsilon > 1$ [14,33,43]. Much like the TSP, polynomial time approximation algorithms with guaranteed performance ratios exist for BTSP on specially structured problem data [6,14,22,26,33]. Moreover, several special cases of the problem can be solved to optimality in polynomial time [26].

Garfinkel and Gilbert discussed a branch and bound based exact algorithm to solve the BTSP and reported computational results with a construction heuristic on randomly generated problems of sizes up to 100 nodes [18]. Timofeev [47] reported experimental results on problems of similar size but with a heuristic algorithm. Sergeev proposed a dynamic programming approach [45] while Carpento et al. reported experimental results with a branch and bound algorithm on problems of size up to 200 nodes [11]. Ramakrishnan et al. presented experimental results with a threshold heuristic on 72 symmetric TSPLIB problems of size up to 783 cities [40] and Ahmad [1] reported experimental results using algorithms based on lexicographic search for symmetric TSPLIB instances with less than 300 cities. In a small computational study with less than 100 cities, Ahmad [2] reported experimental results on asymmetric BTSP instance using a lexicographic search based algorithm. Very recently, LaRusic et al. [29] reported extensive experimental results on the symmetric version of BTSP on almost all available test problems (TSPLIB, Johnson–McGeoch random instances, VLSI and National TSP instances up to 31,623 nodes) and obtained optimal solutions for most of these instances.

In this paper, we focus on the asymmetric version of the BTSP which is not thoroughly investigated in the literature. The best known performance ratio of a polynomial time approximation algorithm for the symmetric TSP with cost matrix satisfying triangle inequality is $\frac{3}{2}$ [12] whereas for the asymmetric TSP it is $O(\log n)$ [17]. Reducing this gap is a well known open problem. In the case of BTSP, it is well known that the symmetric version can be approximated with a performance ratio of 2 whenever the edge weights satisfy the triangle inequality [14,26,33] and this is the best possible bound for a polynomial time algorithm (unless P=NP) for this class of cost matrices. For the asymmetric BTSP, no polynomial time approximation algorithm with bounded

☆This work was supported by NSERC discovery grants awarded to Abraham P. Punnen.

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http://dx.doi.org/10.1016/j.cor.2013.08.005

performance ratio is known even with triangle inequality assumption on the cost matrix. We give a polynomial time approximation algorithm for asymmetric BTSP with performance ratio $\lceil n/2 \rceil$ whenever the edge costs satisfy the triangle inequality, and generalize this result to the case where edge costs satisfying the $\tau-$triangle inequality.

Further, extending the algorithms for the symmetric version reported in [29], we develop a binary search based heuristic for the asymmetric BTSP and report results of extensive computational experiments on all available benchmark test instances for the asymmetric TSP. To the best of our knowledge, no such extensive computational study on asymmetric BTSP is available in the literature. Our algorithm produced optimal solutions for 270 out of 331 problems considered and this is achieved within a very reasonable computational time. We establish optimality certificate in the majority of instances by developing various lower bounding schemes that produce very tight bounds. Extensive theoretical and experimental comparisons of these lower bounds are also given. The optimality of the remaining problems is established by an exact optimization scheme which is obtained by modifying our heuristic algorithm.

The maxmin version of the BTSP is called the *maximum scatter traveling salesman problem* (MSTSP) [5] which is defined as follows:

$$\begin{aligned} &\text{Maximize} \quad \min\{c_{ij} : (i,j) \in H\} \\ &\text{subject to} \quad H \in \Pi(G). \end{aligned} \tag{2}$$

Arkin et al. [5] showed that the symmetric version of MSTSP is NP-Complete, and no constant-factor approximation algorithm exists for the problem unless P=NP. They also provided a 2-approximation algorithm for the MSTSP with a symmetric cost matrix satisfying the triangle inequality and discussed applications of the model in sequencing rivet operations when fastening sheets of metal together in the aircraft industry among others. Kabadi and Punnen [26] obtained a $2\tau-$approximation algorithm for the MSTSP whenever the cost matrix satisfies the $\tau-$triangle inequality and this is the best possible bound for such cost matrices.

The MSTSP can be formulated as a BTSP using the transformation $d_{ij} = M - c_{ij}$ where $D = (d_{ij})_{n\times n}$ is the cost matrix for the equivalent BTSP and M is a sufficiently large number. While this transformation preserves optimality, it does not preserve $\epsilon-$optimality. However, we show that the heuristic developed for the BTSP works reasonably well in practice for the MSTSP under this transformation.

The paper is organized as follows. Section 2 discusses approximation algorithms for the BTSP. In Section 3 we consider lower bounds for the asymmetric BTSP and Section 4 discusses our primary heuristic algorithm, which can easily be modified into an exact BTSP solver. Extensive computational results are reported and discussed in Section 5. Section 6 presents computational results on MSTSP, and concluding remarks are given in Section 7.

For any directed graph G $\delta^+(v)$ and $\delta^-(v)$, respectively, denote the in-degree and the out-degree of vertex v. Since we assume that G is a complete digraph, an instance of BTSP is completely defined by the cost matrix C. For that reason, we use the terminology BTSP on G and BTSP on C interchangeably. Also, for simplicity, a tour in G with cost matrix C is sometimes referred to as a tour in C. A lower bound for a problem means a lower bound for the optimal objective function value of the problem. Finally, for any spanning subgraph S of G, we denote $C_{\max}(S) = \max\{c_{ij} : (i,j) \in S\}$.

2. Approximation algorithms

Approximation algorithms for TSP is a thoroughly investigated research area and the behavior of its symmetric and asymmetric versions are quiet different in terms of approximability. When the edge costs satisfy the triangle inequality, the symmetric version has a $\frac{3}{2}-$approximation algorithm [12] while the best known performance ratio for the asymmetric version is $O(\log n)$ [17,27,28]. The behavior of the BTSP in terms of approximability appears even more intriguing. The symmetric version can be approximated within a factor of 2 whenever the cost matrix satisfies the triangle inequality and this is the best possible performance bound (unless P=NP). For the asymmetric version no $\epsilon-$approximation algorithm is reported in the literature for any $\epsilon > 1$ even if the edge costs satisfy the triangle inequality. It is easy to see that no polynomial time approximation algorithm with a data independent performance ratio exists for BTSP (unless P=NP) on an arbitrary cost matrix [33]. Thus we restrict our attention in this section to asymmetric instances where the edge weights satisfy the $\tau-$triangle inequality. Note that a cost matrix $C = (c_{ij})_{n\times n}$ satisfies *$\tau-$triangle inequality* if $c_{ij} \le \tau(c_{ik} + c_{kj})$ for all $i,j,k \in V$.

The tth *power of a graph* (not necessarily complete) G is the graph $G^t = (V, E^t)$, where $(u,v) \in E^t$ whenever a path from u to v exists in G with at most t edges.

Theorem 1. *Let C be the cost matrix associated with a complete digraph G satisfying $\tau-$triangle inequality for some $\tau > \frac{1}{2}$ and let H^0 be an optimal solution to the BTSP on G. Let S be a spanning subgraph of G such that $C_{\max}(S) \le C_{\max}(H^0)$. If the graph S^t, $1 \le t < n$ and integer t, contains a Hamiltonian cycle H, then*

$$\frac{C_{\max}(H)}{C_{\max}(H^0)} \le \begin{cases} t & \text{if } \tau = 1 \\ \dfrac{\tau}{\tau-1}(2\tau^{t-1} - \tau^{t-2} - 1) & \text{if } \tau > 1 \\ \dfrac{\tau}{\tau-1}(\tau^{t-1} + \tau - 2) & \text{if } \tau < 1 \end{cases}$$

Theorem 1 above was originally proved by Kabadi and Punnen [26] for the symmetric BTSP case. However, the proof is almost identical for the asymmetric version and hence we skip the detailed proof.

Our approximation algorithm was inspired by the 2-approximation algorithm for BTSP on a complete undirected graphs with edge-costs satisfying the triangle inequality discussed in [33,26,14,22]. A formal description of our approximation algorithm for BTSP on a complete directed graph G is given below.

Algorithm Approx-BTSP:

- Step 1: Compute a bottleneck strongly connected spanning subgraph S of G.
- Step 2: Find S^t for $t = \lceil n/2 \rceil$.
- Step 3: Output any hamiltonian cycle in S^t.

To establish the complexity and performance ratio of algorithm Approx-BTSP we use the following well known theorem of Ghouilà-Houri [19].

Theorem 2 (*Ghouilà-Houri [19]*). *If G is a directed graph on n vertices and $\min\{\delta^+(v), \delta^-(v)\} \ge n/2$ for every vertex $v \in G$, then G is Hamiltonian.*

Theorem 3. *Algorithm Approx-BTSP runs in polynomial time and guarantees an $\epsilon-$optimal solution for the asymmetric BTSP whenever the edge-costs satisfy the $\tau-$triangle inequality, where*

$$\epsilon = \begin{cases} \left\lceil \dfrac{n}{2} \right\rceil & \text{if } \tau = 1, \\ \dfrac{\tau}{\tau-1}(2\tau^{\lceil n/2 \rceil - 1} - \tau^{\lceil n/2 \rceil - 2} - 1) & \text{if } \tau > 1, \\ \dfrac{\tau}{\tau-1}(\tau^{\lceil n/2 \rceil - 1} + \tau - 2) & \text{if } \tau < 1. \end{cases}$$

Proof. Let H^0 be an optimal solution to the BTSP on G. Since S is a strongly connected spanning subgraph of G, we have $C_{\max}(S) \le C_{\max}(H^0)$. Thus by Theorem 1, the performance ratio holds. We now