Computers & Operations Research 43 (2014) 68–77

Contents lists available at ScienceDirect

Computers & Operations Research

journal homepage: www.elsevier.com/locate/caor

Computation and application of the paired combinatorial logit stochastic user equilibrium problem

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ARTICLE INFO

Available online 29 August 2013

Keywords:
Logit
Paired combinatorial logit
Stochastic user equilibrium
Traffic assignment
Partial linearization
Line search

ABSTRACT

The paired combinatorial logit (PCL) model is one of the recent extended logit models adapted to resolve the overlapping problem in the route choice problem, while keeping the analytical tractability of the logit choice probability function. However, the development of efficient algorithms for solving the PCL model under congested and realistic networks is quite challenging, since it has large-dimensional solution variables as well as a complex objective function. In this paper, we examine the computation and application of the PCL stochastic user equilibrium (SUE) problem under congested and realistic networks. Specifically, we develop an improved path-based partial linearization algorithm for solving the PCL SUE problem by incorporating recent advances in line search strategies to enhance the computational efficiency required to determine a suitable stepsize that guarantees convergence. A real network in the city of Winnipeg is applied to examine the computational efficiency of the proposed algorithm and the robustness of various line search strategies. In addition, in order to acquire the practical implications of the PCL SUE model, we investigate the effectiveness of how the PCL model handles the effects of congestion, stochasticity, and similarity in comparison with the multinomial logit stochastic traffic equilibrium problem and the deterministic traffic equilibrium problem.

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1. Introduction

The stochastic user equilibrium (SUE) principle was suggested by Daganzo and Sheffi [21] more than 30 years ago to relax the perfect knowledge assumption of network travel times of the deterministic user equilibrium (DUE) model. It is defined as follows:

> "*At SUE, no motorists can improve his or her perceived travel time by unilaterally changing routes*" [21].

Specifically, a random error term is incorporated in the route choice decision process to simulate travelers' imperfect perceptions of network travel times, such that they do not always end up picking the minimum travel time route. The random error term here is interpreted as the perception error of network travel times due to the travelers' imperfect knowledge of network conditions. In this model, each traveler is assumed to have some perceptions of the mean travel times on each link of the network. Each traveler's route choice criterion is to minimize the *perceived* value of the route travel time, which can be obtained by adding up the *perceived* travel times on all the links belonging to the route.

Route choice models proposed under this approach can have different specifications according to modeling assumptions on the random error term. The two commonly used random error terms are Gumbel [23] and Normal [21] distributions, which result in the logit- and probit-based route choice models, respectively. The logit-based route choice model has a closed-form probability expression and also an equivalent mathematical programming (MP) formulation for the SUE problem under congested networks [25]. The multinomial logit (MNL) SUE MP formulation can be solved using both path enumeration techniques [9,11,46] and column (or path) generation techniques [7,8,12–13,16,18–19,22,27,30,33]. Column generation techniques for the MNL SUE model can be implemented in the link- or path-based domains. The link-based algorithms [7,18,27,33] do not require path storage and often use Dial's STOCH algorithm [23] or Bell's alternative [6] as the stochastic loading step, while the path-based algorithms [8,12–13,16,19,22,30,46] require explicit path storage in order to directly compute the logit route choice probabilities. The drawbacks of the logit model are: (1) inability to account for overlapping (or correlation) among routes, and (2) inability to account for perception variance with respect to trips with different lengths. These two drawbacks stem from its underlying assumptions that the random

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0305-0548/$ - see front matter Published by Elsevier Ltd.
http://dx.doi.org/10.1016/j.cor.2013.08.022

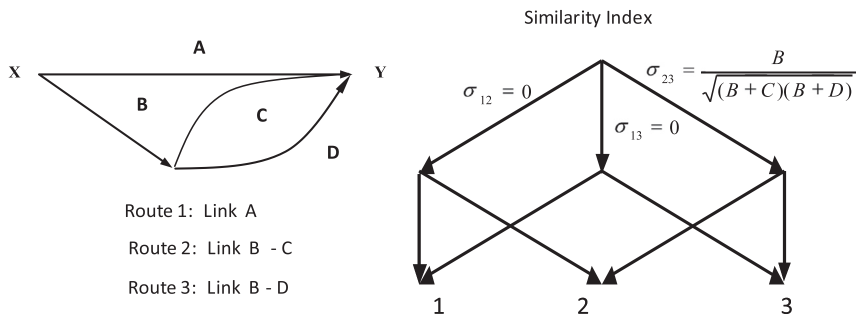

Fig. 1. Hierarchical structure of the paired combinatorial logit model.

error terms are independently and identically distributed (IID) with the same and fixed variances [43]. On the other hand, the probit-based route choice model does not have such drawbacks, because it handles the overlapping and identical variance problems between routes by allowing covariance between the random error terms for pairs of routes. However, the probit model does not have a closed-form solution and it is computationally burdensome to solve the multiple integrals when the choice set contains more than a handful of routes. Due to the lack of a closed-form probability expression, solving the probit-based SUE model will require either Monte Carlo simulation [44], Clark's approximation method [32,34], or numerical method [42].

Recently, there are renewed interests to relax the assumptions of the MNL SUE model. Several modifications or generalizations of the logit structure have been proposed to relax the IID assumptions of the MNL SUE model (see [38,39] for a comprehensive review of these recent advances of the logit model). Among these extended logit models, the paired combinatorial logit (PCL) model is considered the most suitable for adaptation to the route choice problem to resolve the overlapping problem while keeping the analytical tractability of the logit choice probability expression. In the PCL model, each pair of alternatives can have a similarity relationship that is completely independent of the similarity relationship of other pairs of alternatives. Bekhor and Prashker [2] argued that this feature is highly desirable for route choice models, since each pair of routes may have different similarities. Further, Gliebe et al. [26] demonstrated that the PCL model could be scaled to account for perception variance with respect to different trip lengths. Both features are useful in addressing the well-known "independent of irrelevant alternatives" (IIA) property inherited in the MNL model. Pravinvongvuth and Chen [40] examined these two features using simple examples (i.e., three routes in the loop-hole network) with limited number of routes to illustrate the flexibility of PCL's hierarchical tree structure in overcoming the drawbacks of the MNL model. The nice features of the PCL model have been theoretically explored and numerically demonstrated. However, there has been quite less development of efficient algorithms for solving the PCL model under congested and realistic networks. The computationally complexity mainly comes from the large-dimensional solution variables as well as the complex objective function when implementing in large-scale realistic networks.

In this paper, we examine the computation and application of the PCL SUE problem under congested and realistic networks. Specifically, we develop an improved path-based partial linearization algorithm for solving the PCL SUE mathematical programming formulation by incorporating recent advances in line search strategies. The partial linearization method is a descent algorithm for solving convex optimization problems [37]. It iterates between the search direction and line search steps until some convergence criterion is reached. The search direction is obtained by solving a first-order approximation of an additive part of the PCL SUE objective function (to be shown later in Section 3). For the line search step, recent advances in line search strategies (e.g., self-regulated averaging (SRA) scheme and quadratic interpolation scheme) and the traditional line search methods (e.g., method of successive averages (MSA), bisection, and Armijo) are examined in order to efficiently determine a suitable stepsize. A real-size network in the city of Winnipeg is applied to examine the efficiency and robustness of various line search schemes that minimize the computational efforts required to determine a suitable stepsize while guaranteeing convergence. In addition, the practical implications of implementing the PCL SUE model are acquired by examining the effects of congestion, stochasticity, and similarity in comparison with the multinomial logit stochastic traffic equilibrium problem and the deterministic traffic equilibrium problem.

The paper is organized as follows. The PCL route choice model and the equivalent PCL SUE formulation are presented in Section 2. The path-based partial linearization algorithm with various line search schemes is described in Section 3. Computational results and practical implications are then presented in Section 4 and Section 5, respectively. Finally, some conclusions are summarized in Section 6.

2. Paired combinatorial logit stochastic user equilibrium problem

2.1. PCL route choice model

The paired combinatorial logit (PCL) model was originally proposed by Chu [20], further developed by Koppleman and Wen [29] to examine the structure, properties, and estimation, and adapted to model route choice decisions by Bekhor and Prashker [2], Gliebe et al. [26], and Pravinvongvuth and Chen [40]. In contrast to the simple structure of the logit model, the PCL model has a hierarchical structure that decomposes the choice probability into two levels represented by the marginal and conditional probabilities (see Fig. 1). Thus, the PCL choice probability can be expressed as

$$P(k) = \sum_{j \neq k} P(kj) \cdot P(k|kj), \tag{1}$$

where

$$P(k|kj) = \frac{e^{V_k/(1-\sigma_{kj})}}{e^{V_k/(1-\sigma_{kj})} + e^{V_j/(1-\sigma_{kj})}}, \tag{2}$$

$$P(kj) = \frac{(1-\sigma_{kj})(e^{V_k/(1-\sigma_{kj})} + e^{V_j/(1-\sigma_{kj})})^{1-\sigma_{kj}}}{\sum_{l=1}^{n-1}\sum_{m=l+1}^{n}(1-\sigma_{lm})(e^{V_l/(1-\sigma_{lm})} + e^{V_m/(1-\sigma_{lm})})^{1-\sigma_{lm}}}, \tag{3}$$

where V_k is the observable component of the utility for alternative k (i.e., $U_k = V_k + \varepsilon_k, \quad \forall \ k$); σ_{kj} is a similarity index between alternatives k and j; and n is the number of alternatives. $P(k|kj)$ is the conditional probability of choosing alternative k given that the alternative pair kj has been chosen, and $P(kj)$ is the marginal (unobserved) probability