Computers & Operations Research 43 (2014) 78–89

Contents lists available at ScienceDirect

Computers & Operations Research

journal homepage: www.elsevier.com/locate/caor

Generalized quadratic multiple knapsack problem and two solution approaches

Tugba Saraç*, Aydin Sipahioglu

Industrial Engineering Department of Eskisehir Osmangazi University, Meselik, 26480 Eskisehir Turkey

ARTICLE INFO

Available online 31 August 2013

Keywords:
Generalized Quadratic Multiple Knapsack Problem (G-QMKP)
F-MSG
Genetic Algorithm (GA)
Production with plastic injection
Combinatorial optimization

ABSTRACT

The Quadratic Knapsack Problem (QKP) is one of the well-known combinatorial optimization problems. If more than one knapsack exists, then the problem is called a Quadratic Multiple Knapsack Problem (QMKP). Recently, knapsack problems with setups have been considered in the literature. In these studies, when an item is assigned to a knapsack, its setup cost for the class also has to be accounted for in the knapsack. In this study, the QMKP with setups is generalized taking into account the setup constraint, assignment conditions and the knapsack preferences of the items. The developed model is called Generalized Quadratic Multiple Knapsack Problem (G-QMKP). Since the G-QMKP is an NP-hard problem, two different meta-heuristic solution approaches are offered for solving the G-QMKP. The first is a genetic algorithm (GA), and the second is a hybrid solution approach which combines a feasible value based modified subgradient (F-MSG) algorithm and GA. The performances of the proposed solution approaches are shown by using randomly generated test instances. In addition, a case study is realized in a plastic injection molding manufacturing company. It is shown that the proposed hybrid solution approach can be successfully used for assigning jobs to machines in production with plastic injection, and good solutions can be obtained in a reasonable time for a large scale real-life problem.

1. Introduction

The knapsack problem (KP) is a well-known combinatorial optimization problem. The classical KP seeks to select, from a finite set of items, the subset, which maximizes a linear function of the items chosen, subject to a single capacity constraint. In many real life applications, it is important that the profit of packing should also reflect how well the given items fit together. One formulation of such interdependence is the quadratic knapsack problem (QKP). The QKP asks to maximize a quadratic objective function subject to a single capacity constraint. Portfolio management problems, the determination of the optimal sites for communication satellite earth stations or railway stations are good examples of the QKP [8]. The QKP was introduced and solved using a branch-and-bound algorithm by Gallo, Hammer and Simeone in 1980. Later, different branch and bound based solution techniques were offered by Chaillou, Hansen and Mahieu [7], by Michelon and Veuilleux [22] and by Billionnet and Calmels [3]. An exact algorithm was developed by [5] and by Billionnet and Soutif in 2003. In 2005, a greedy Genetic Algorithm was proposed by Julstrom. A survey of upper bounds presented in the literature has been given, and the relative tightness of several of the bounds has been shown by Pisinger [23]. In the same year, Xie and Liu [30] presented a mini-swarm approach for the QKP. In 2009, Sipahioglu and Saraç examined the performance of the modified subgradient algorithm (MSG) to solve the 0-1 QKP and they showed that the MSG algorithm can be successfully used for solving the QKP.

The Quadratic multiple knapsack problem (QMKP) extends the QKP with k knapsacks, each with its own capacity c_k. Hiley and Julstrom [13] proposed the first study regarding QMKP in the literature. The paper introduced three heuristic approaches, namely the greedy heuristic, the stochastic hill-climber and the Genetic Algorithm (GA). The greedy heuristic fills the knapsacks one item at a time, always choosing the unassigned item with the highest profit/weight ratio of values to other items with a weight smaller than the remaining capacity of the knapsack. The hill-climber's neighbor operator removes objects from each knapsack, and then refills the knapsack greedily as in the greedy heuristic. The hill-climber's neighbor operator also serves as the GA's mutation. Saraç and Sipahioglu [25] proposed a hybrid genetic algorithm to solve the QMKP. They developed a specialized crossover operator to maintain the feasibility of the chromosomes and presented two distinct mutation operators with different improvement techniques from the non-evolutionary heuristic. They also showed that their GA was more successful than the GA presented by Hiley and Julstrom [13], especially in the case where the number of knapsacks (k) increases. In 2007, Singh and Baghel proposed a steady-state grouping genetic algorithm for the QMKP. Like Hiley and Julstrom [13], they also

* Corresponding author. Tel.: +90 2222393750.
E-mail addresses: tsarac@ogu.edu.tr, tugba.sarac@gmail.com (T. Saraç), asipahi@ogu.edu.tr (A. Sipahioglu).

0305-0548/$ - see front matter © 2013 Elsevier Ltd. All rights reserved.
http://dx.doi.org/10.1016/j.cor.2013.08.018

assumed that all the knapsack capacities c_k where the same. They compared their results with two previously proposed methods; the genetic algorithm and the stochastic hill climber [13]. The results show the effectiveness of their approach. The average solution values obtained were always better than those obtained with the genetic algorithm and the stochastic hill climber. In 2010, [28] proposed a variant of the artificial bee colony algorithm for the QMKP. Their computational results show the superiority of their approach over other approaches in terms of solution quality.

In recent years, studies by McLay and Jacobson [20], Caserta et al. [6], Altay et al. [1], Michel et al. [21] on the knapsack problem that takes setup constraints into consideration have begun to be included in the literature. In these problems, when an item is assigned to a knapsack, the setup cost for its class also has to be attained to the knapsack. Further, not only the weight of the item, but also the cost for the setup has to be taken into consideration in terms of capacity utilization. In the literature, items that required a common setup, rather than separate setups, when being assigned to the same knapsack, were considered as items of the same class or family. McLay and Jacobson [20] provide three dynamic programming algorithms that solve the Bounded Setup Knapsack Problem (BSKP) in pseudo-polynomial time and a fully polynomial-time approximation scheme (FPTAS). One of the dynamic programming algorithms presented solves the Bounded Knapsack Problem (BKP) with the same time and space bounds of the best known dynamic programming algorithm for the BKP. The FPTAS improves the worst-case time bound for obtaining approximate solutions to the BKP compared to using FPTASs designed for the BKP or the 0-1 Knapsack Problem. Caserta et al. [6], proposed a new meta-heuristic based algorithm for the integer knapsack problem with setups. The proposed algorithm is a cross entropy based algorithm, where the meta-heuristic scheme allows a relaxation of the original problem to a series of well-chosen standard knapsack problems, solved through a dynamic programming algorithm. Altay et al. [1], considered a class of knapsack problems that included setup costs for families of items. A mixed integer programming formulation for the problem was provided along with exact and heuristic solution methods. The computational performances of the algorithms were reported and compared with CPLEX. Michel et al. [21] considered multiple-class integer knapsack problem with setups. Their paper provides a review of the literature on knapsack problems with setups, discusses various reformulations, and presents specialized branch-and-bound procedures extending the standard algorithm for the knapsack problem. Sang and Sang [24], proposed a new memetic algorithm for the quadratic multiple container packing problem. The proposed memetic algorithm is based on the adaptive link adjustment evolutionary algorithm (ALA-EA) and it incorporates heuristic fitness improvement schemes into the ALA-EA. Wang et al. [29], provided a comparison of quadratic and linear representations of the QKP based on test problems with multiple knapsack constraints and up to eight hundred variables. In addition to the setup constraints, there may be other important conditions such as the knapsack preference of the items.

In this study, the QMKP with setups is generalized by considering the knapsack preferences of the items and is called the Generalized Quadratic Multiple Knapsack Problem (G-QMKP). It appears to be, it is the first study on generalized quadratic multiple knapsack problems in the literature.

KP that are combinatorial optimization problems belong to the class of NP-hard type problems [18]. Both the QMKP and the G-QMKP are NP-hard by restriction to KP; even if all the quadratic values p_{ij} are set equal to zero and the number of knapsacks equal one. Since solving the G-QMKP is not easy, an efficient search heuristic approach will be useful for solving this problem.

Genetic algorithms are powerful and broadly applicable in stochastic search and optimization techniques based on principles from evolution theory [11]. GAs, which differ from normal optimization and search procedures: (a) work with a coding of the parameter set, not the parameters themselves; (b) search from a population of points, not a single point; (c) use payoff (objective function) information, not derivatives or other auxiliary information; and (d) use probabilistic transition rules, not deterministic rules [12]. Due to these characteristics, use of the GA in numerous fields has been rapidly increasing in the recent years. The GA has also been successfully implemented in the QKP [15] and the QMKP [13,25,26] problems. Another solution method that comes into prominence with its success in solving integer nonlinear problems in recent years [27] is based on the Modified Subgradient (MSG) algorithm. The MSG algorithm was proposed by Gasimov [9] for solving dual problems constructed in respect to a sharp augmented Lagrangean function. If the problem is not convex, using classical Lagrangean based solution methods may lead to a non-zero duality gap. To eliminate this problem, a sharp augmented Lagrangean function is used to construct the dual problem in the MSG algorithm. It is proven that, when the objective and constraint functions are all Lipschitz, then the sharp augmented Lagrangean guarantees the zero duality gap [9]. The MSG algorithm also has some outstanding properties. For example, the MSG algorithm is convergent and does not require any convexity or differentiability conditions on the primal problem. Furthermore, it does not use any penalty parameters, and guarantees the strict increment of dual values at each iteration. Nevertheless, the MSG algorithm also has some disadvantages. It is necessary to find the global optimum of the unconstraint problem at every iteration and uses an upper limit for updating step size parameters. To cope with these problems, the F-MSG algorithm was introduced by [16]. The F-MSG algorithm was used to solve the Quadratic Assignment Problem (QAP) and very good results were obtained [10]. However, the performance of the F-MSG algorithm still depends on the performance of the solution method used for solving the sub problem at any iteration.

In this study, firstly a mathematical model for the G-QMKP is developed and then two different meta-heuristic solution approaches are proposed to solve it. The first method is a genetic algorithm based solution approach and the second method is a hybrid solution approach, which combines the F-MSG (modified subgradient algorithm based on feasible values) and a genetic algorithm for solving the sub problem. Additionally, the success of the proposed methods is shown and compared with the results obtained using the Gams/Dicopt solver on randomly generated test instances. Finally, a case study is realized on a plastic injection molding company as an implementation of the proposed approaches. In this implementation, a large scale real world problem is solved, and it is shown that the proposed hybrid solution approach can be used successfully for assigning jobs to machines in plastic injection production. Moreover, good solutions can be obtained in a reasonable time for a large scale real-life problem.

The organization of this paper is as follows. The second section gives a mathematical model for the G-QMKP. The third section describes the proposed solution methods for both the genetic algorithm and the hybrid algorithm, in detail. Computational results are presented in Section 4. A case study is given in Section 5 and, finally, the conclusions are outlined in Section 6.

2. Mathematical model of the generalized quadratic multi knapsack problem

The mathematical model of the Quadratic Multiple Knapsack Problem is as follows:
Index sets:

$J=\{j|j=1,\dots,n\}$ index set of items
$K=\{k|k=1,\dots,m\}$ index set of knapsacks