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The vehicle rescheduling problem

Remy Spliet*, Adriana F. Gabor, Rommert Dekker

Econometric Institute, Erasmus University Rotterdam, P.O. Box 1738, 3000 DR Rotterdam, The Netherlands

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ABSTRACT

The capacitated vehicle routing problem (CVRP) is the problem of finding a routing schedule to satisfy demand by supplying goods stored at the depot, such that the traveling costs are minimized. For operational purposes, in many practical applications a long term routing schedule is made, often based on average demand. When demand substantially differs from the average, constructing a new schedule is beneficial. The vehicle rescheduling problem (VRSP) is the problem of finding a new schedule that not only minimizes the total traveling costs but also minimizes the costs of deviating from the original schedule. In this paper a mathematical programming formulation of the rescheduling problem is presented as well as a heuristic solution method referred to as the two-phase heuristic. We provide sufficiency conditions for which it produces the optimal solution. Finally, we perform computational experiments to study the performance of the two-phase heuristic.

1. Introduction

1.1. Scheduling and rescheduling

The capacitated vehicle routing problem (CVRP) is a classical problem in operations research. Consider a depot where goods are stored and a set of customers that have nonnegative demand for these goods. A set of homogeneous vehicles of finite capacity is available to transport the goods from the depot to the customers. The vehicles start and end their routes at the depot. Costs are incurred for traveling from one location to another. The CVRP is to find a routing schedule that describes the sequence of locations visited by every vehicle that minimizes the total traveling costs, while the capacity constraints are satisfied. The CVRP is known to be an NP-hard problem.

Many solution methods can be found in the scientific literature for the CVRP. The branch-and-cut scheme of Baldacci et al. [1] seems presently to be one of the most successful at solving CVRP instances of up to 100 customer locations. For larger problem instances, many heuristic algorithms have also been developed that are able to find good solutions with greater speed. An overview of exact and heuristic algorithms can be found in Fisher [2], Toth and Vigo [3], Laporte [4,5] and Laporte et al. [6] amongst others.

In the classical CVRP, demand is deterministic and known. A situation that often occurs in practice is that demand only becomes apparent at a late moment. For example, in the retail industry it is very common that the orders of the individual stores are placed only a few days, sometimes even just one day, before delivery. In these situations it is beneficial for operational processes to determine the delivery schedule before the orders are placed. It is for instance very costly, if at all possible, to make a roster for delivery handling personnel shortly before they are needed. A common solution to this problem is to determine a long term schedule, henceforward *master schedule*, that serves as a guiding schedule over a certain period of time in which multiple deliveries are made. For example, such a master schedule would describe the weekly or even daily deliveries for a period of six months. In practice, master schedules are usually constructed by solving deterministic CVRP instances based on average customer demand as predicted for the upcoming period. In this paper we assume that such a master schedule is given.

A master schedule is thus made before demand realizations become apparent. As a result, when the demand becomes known, the master schedule may not be optimal due to inefficient use of the vehicles, or may even be infeasible due to violation of the capacity constraints. In such cases the master schedule needs to be deviated from. This is often necessary in practice, for example when demand of customers is highly correlated, as typically is the case in a retail chain. The construction of a new schedule, when demand realizations become known, will be referred to as *rescheduling*.

1.2. Effects of rescheduling

After rescheduling, the new schedule will typically deviate from the master schedule. This can have negative effects on a distribution network. Locations are visited in a different order, by different trucks or by different truck drivers than initially planned. This may cause confusion among drivers and negatively affect the

* Corresponding author.
E-mail address: Spliet@ese.eur.nl (R. Spliet).

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regularization and personalization of service, as is also recognized by Bertsimas and Simchi-Levi [7] and Li et al. [8,9]. Furthermore, consider the situation where personnel is hired only for handling deliveries and deliveries do not arrive at the agreed upon moment due to a deviation in the schedule. Here, labor costs increase due to the fact that personnel has to work overtime or has to be hired for another shift. When rescheduling is done by constructing a completely new schedule, many deviations may occur resulting in high additional costs.

Our experience with Dutch retail companies has shown that currently rescheduling is often done manually. Dispatchers typically operate under the notion that when a route needs to be deviated from, costs are lower when the first deviation occurs at a later stage in the route. There are several arguments that support this notion. Firstly, when deviating at a late stage in each route large portions of the master schedule remain intact, diminishing the above-mentioned negative effects of rescheduling. Secondly, the changes made in this manner are easily communicated through the distribution network. Finally, when changing the first locations of a route, the dispatching times of the truck will be altered. However, changing the working hours of a driver at a late moment is very expensive and often practically infeasible.

1.3. The vehicle rescheduling problem

In this paper, we propose a rescheduling model in which the negative effects of deviating from the master schedule are incorporated. We introduce deviation costs, which are incurred for each route that deviates from the master schedule. Furthermore, the height of the deviation cost per route is dependent on the customer at which the first deviation occurs and the position in the route it has. In this way, we are able to model the above described notion of dispatchers that deviations early in a route are more costly than deviations late in a route.

Given a master schedule and a demand realization, the goal is to find a new schedule that minimizes the total traveling and deviation costs, while satisfying the capacity constraints. This problem will be referred to as the vehicle rescheduling problem (VRSP). This model is of particular interest to for instance retail chains that control both the supply chain and the stores, as they not only incur the transportation costs, but also both the deviation costs at the supply side and at the customer side.

1.4. Rescheduling in current literature

In the literature, rescheduling is mainly considered in conjunction with designing a master schedule. Given a rescheduling method, the master schedule is designed before demand is known such that the expected costs incurred after rescheduling are minimized. The rescheduling method proposed by Bertsimas [10] is maybe the most popular method in the literature. In this method, the master schedule is used until a vehicle arrives at a location where its cargo is depleted. After it has returned to the depot to refill, the vehicle resumes the master schedule from the location where it left off. Under this rescheduling protocol, for specific demand distributions the expected costs of a master schedule can easily be calculated. For this reason, the rescheduling method proposed by Bertsimas has been incorporated in models used to design a master schedule with minimal expected costs. Examples of solution methods to solve these models are the L-Shaped integer method to find the optimal master schedule by Laporte et al. [11], a tabu search heuristic by Gendreau et al. [12], a rollout algorithm by Secomandi [13] and an evolutionary algorithm by Tan et al. [14].

In a study by Groër et al. [15] they propose to reschedule in such a way that each customer is always visited by the same driver and within the same time window. They apply this to a setting encountered in the small-package shipping industry in which a customer does not require service on all delivery days. In their paper they focus on generating a master schedule for large instances and do this using a local search heuristic. Similarly, Chen et al. [16] consider an arc-routing model for small-package delivery in which arcs that do not need service are skipped.

To the best of our knowledge, the literature on rescheduling strategies that take into account deviation costs is scarce. Li et al. [8,9] consider the problem of reassigning vehicles to trips, when one of the vehicles breaks down. In their model, costs are incurred when trips are delayed. The main application of this model is in passenger transportation, for situations where traveling costs and capacity constraints do not play an important role.

1.5. Contribution

In this paper, we introduce a novel model for the rescheduling problem which can be applied where demand is known shortly before vehicles are dispatched. We provide a mixed integer programming formulation based on a formulation of the CVRP by Baldacci et al. [1]. Using this formulation, the VRSP of moderate size can be solved by general purpose optimization software or slight modifications of existing algorithms for solving the CVRP.

Furthermore, we design a solution approach based on first removing the last locations of routes and rescheduling them. We will refer to this approach as the *two-phase heuristic*. We analyze the performance of this heuristic and derive sufficiency conditions on the value of the deviation costs for which the two-phase heuristic is guaranteed to give the optimal solution to the VRSP. Moreover, numerical experiments indicate that, in general, for low deviation costs the two-phase heuristic often provides optimal or near optimal solutions. Finally, we describe this algorithm in such a way that it can be implemented directly in existing commercial CVRP software available to many dispatchers in large distribution networks.

1.6. Outline

In the following section, the VRSP is described in detail and a mixed integer linear programming formulation is presented. In Section 3, the two-phase heuristic is presented. It is accompanied by an analysis of its behavior with respect to the deviation costs. Finally, in Section 4 the sensitivity of the solution to the VRSP with respect to the value of the deviation costs is investigated and the performance of the two-phase heuristic is studied by comparing its solutions with the optimal solutions of the VRSP.

2. The vehicle rescheduling problem formulation

In this section, first the vehicle rescheduling problem is defined. This is followed by a mixed integer programming formulation based on an existing model developed by Baldacci et al. [1]. In Section 4, we will use this MIP formulation in computational experiments.

2.1. Problem definition

Consider an undirected complete graph $G=(V,E)$. The set of nodes $V=\{0,1,\dots,n+1\}$ corresponds to a starting depot 0, an ending depot $n+1$ and the set of customers $V'=\{1,\dots,n\}$. For every edge $(i,j)\in E$, traveling costs $c_{ij}\geq 0$ are given that satisfy the triangle inequality. We suppose that an unlimited number of vehicles of capacity Q are available for supplying goods to the