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Optimizing the strategic patient mix combining queueing theory and dynamic programming

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ABSTRACT

In this paper we address the decision of choosing a patient mix for a hospital that leads to the most beneficial treatment case mix. We illustrate how capacity, case mix and patient mix decisions are interrelated and how understanding this complex relationship is crucial for achieving the maximum benefit from the fee-for-service financing system. Although studies to determine the case mix that is of maximum benefit exist in the literature, the hospital actions necessary to realize this case mix have seen less attention. We model the hospital as an $M/G/\infty$ queueing system to evaluate the impact of accepting certain patient types. Using this queueing model to generate the parameters, an optimization problem is formulated. We propose two methods for solving the optimization problem. The first is exact but requires an integer linear programming solver whereas the second is an approximation algorithm relying only on dynamic programming. The model is applied in the department of surgery at a Dutch hospital. The model determines which patient types result in the desired growth in the preferred surgical treatment areas. The case study highlights the impact of striving for a certain case mix without providing a sufficiently balanced supply of resources. In the case study we show how the desired case mix can be better achieved by investing in certain capacity.

1. Introduction

In recent years hospital financing has changed from a budget oriented (lump sum) system to a fee-for-service system in many jurisdictions [7]. This transformation is intended to enhance accountability and to motivate hospitals to become more efficient. Diagnosis Related Groups (DRGs), a concept which makes health care services a commodity, are facilitating this change. A DRG describes the whole spectrum of activities involved in treating a certain disease or condition. The reimbursement to the hospital for each DRG treatment is fixed, meaning hospitals that provide the treatment for lower costs can realize greater profits, hence DRGs motivate efficiency. Variants of DRGs were introduced to achieve the same hospital financing transformation in many countries [24].

As a consequence of fee-for-service financing, hospitals must consider their DRG case mix and evaluate which services should be expanded and which should be discontinued. Details of this evolution in the Dutch context are reviewed in [26,13]. An empirical review of hospital behavior in the United States [9] found that hospitals intensify their offerings to high paying procedural DRGs. The majority of papers that investigate financing models and hospital decisions use statistical approaches [15] and draw conclusions about hospital behavior by looking retrospectively. Few models support hospitals in selecting their DRG case mix in the first place. One approach, presented in [23], uses system-dynamics to model a hospital's behavior under the influence of performance-based reimbursement schemes (i.e. fee-for-service DRG schemes). The model allows hospital managers to analyze decisions that will maximize reimbursements. Once a DRG case mix is decided upon, strategies for achieving it must be developed. To our knowledge, this has not been addressed in the literature. Furthermore, empirical research indicates that hospitals struggle to make choices that lead to desired DRG case mixes [4,5].

To achieve a desired DRG case mix, hospitals must entice certain patients to the hospital. Patients are usually referred to the hospital by a general practitioner (GP) who evaluates a patient's symptoms and decides whether the patient should see a specialist. A referral from a GP does not specify which DRG treatment is required but rather the symptoms and the most appropriate modality. The patient then meets a specialist who decides on a treatment plan and based on this treatment plan the corresponding DRGs are recorded. These DRGs may or may not be the ones of greatest benefit, however it is atypical to turn patients away at this point.

Through advertising and promotion to GPs, hospitals can encourage patients with certain diseases or symptoms to come to their

Abbreviations: ASA, approximate solution approach; DRG, diagnosis related group; ILP, integer linear program; NCI, Netherlands Cancer Institute – Antoni van Leeuwenhoek Hospital; PSP, project sequencing problem

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hospital for treatment. However, knowing which symptoms will lead to the desired DRG case mix is not immediately obvious. Arrivals of patients (characterized by their symptoms) follow a stochastic process, and the required treatment cannot be predicted with certainty. Determining, on the basis of symptoms, which types of patients (patient mix) to entice to the hospital in order to achieve the desired DRG case mix is the focus of this paper.

As an example, consider the treatment of colorectal cancer. A patient suspected of having colorectal cancer is referred to a hospital for diagnostic testing. The results from the testing could lead to surgery, chemotherapy, radiotherapy, etc. Within each of the treatment scenarios, there are several treatment options (i.e. DRGs) of which some are more desirable than others. Patient types in this example can be defined in many ways, but common factors indicating the prevalence of colorectal cancer include personal or family history of colorectal cancer and/or bowel disease, ethnic background, diet, weight, etc. Patient types can be further defined by symptoms such as constipation, diarrhea, jaundice, etc. Patient types have uncertain arrival rates and with some probability require specific treatments. Thus choosing the best patient types to achieve the hospital's desired DRG case mix is not immediately obvious.

Hospitals are also constrained by their capacity levels which presumably relate to their desired DRG case mix. When capacity is overwhelmed by an increase in the number of patients, resources become more highly utilized, but patient access times become worse. In this research, to account for quality degradation due to demand exceeding capacity, we limit the fraction of time demand which is allowed to exceed capacity.

In this paper, we choose which patient types lead to a DRG case mix of maximum benefit over time. The chosen patient types are then "added" to the patient mix. Once added, patient types cannot be removed in future periods, as allowing such an "on-again, off-again policy" would create undesired confusion about the offerings of the hospital. In this way, our problem has properties similar to the project sequencing problem (PSP). The PSP determines which capacity expansion projects to implement in order to fulfill a growing demand for capacity.

We model the hospital as an $M_t/G/\infty$ queueing system and formulate an integer linear program (ILP) to exactly solve our problem. Using results from the PSP literature, we also formulate an approximate solution.

Statement of contribution: We develop a mathematical model to determine the policy for accepting new patient types that best matches the desired DRG case mix. To our knowledge it is the first time that capacity, DRG case mix and patient mix decisions are accounted for in a single model to facilitate joint decision making over a long term planning horizon.

The paper is organized as follows. Section 2 formally defines and specifies the optimization problem and the queueing model. Section 3 introduces the PSP and illustrates how it can be used to approximately solve our problem. In Section 4 a case study is solved and the approximation is evaluated. Throughout the paper the terms DRG and treatment are used interchangeably.

2. Model description

The problem addressed in this paper is as follows. Given that a hospital desires a certain DRG case mix, which patient types should be accepted (and when) to maximize a given reward function while ensuring capacity restrictions are accounted for. We assume that the relative importance of the DRGs are known and the capacity of the hospital to provide treatments is known for a finite time into the future. After a patient type is accepted, the number of arrivals of that patient type is modeled as a stochastic process. Upon arrival, a patient of a given type receives treatments according to some given probability distribution. Our model treats time as continuous and considers a finite planning horizon.

The formal problem description and the resulting combinatorial optimization problem are presented in Section 2.1. The calculation of some of the parameters of this combinatorial optimization problem is done using a queueing model which is described in Section 2.2. In Section 2.3 model characteristics are discussed which lead to the discrete time formulation presented in Section 2.4. Finally the complexity of the problem and the motivation for a heuristic solution approach are presented in Section 2.5.

2.1. Combinatorial optimization problem

Consider a set of patient types $P=\{1,2,\ldots,N\}$ and a set of possible treatments $\{1,2,\ldots,M\}$. A patient of type $n\in P$ has a probability $p_{n,m}$ of requiring treatment $m\in\{1,2,\ldots,M\}$. The duration of treatment m has cumulative distribution $B_m(\cdot)$ with mean $\mathbb{E}[B_m]$. Let the number of arrivals of patient type n in period $[0,t)$ be specified by a given random variable $\Lambda_n(t)$ and let $G_m(t)$ be a given model input which describes the volume of ongoing treatments m for which the hospital has the capacity at time t.

For modeling the problem, we introduce variables $S_m(t)$ and $D_m(t)$ where $S_m(t)$ is the distribution for the number of patients receiving treatment m at time t and $D_m(t)$ is the distribution for the number of completed treatments m at time t. Note that $S_m(t)$ and $D_m(t)$ result form the choice of patient types to be accepted. The desired DRG case mix is reflected by values w_m, which specify the relative importance (or value) of treatment m.

The problem now is to indicate for each patient type the first moment in time t_n that patient type n is accepted. Note that for all times after t_n patient type n must also be accepted. Then the goal is to determine t_n such that the weighted number of treatments (weighted according to w_m) is maximized while ensuring that the number of treatments does not exceed $G_m(t)$ for more than a fraction φ_m of time. In other words, a hospital with capacity $G_m(t)$ wishes to maximize the weighted number of treatments they perform, whereby it is acceptable to exceed their capacity fraction $(1-\varphi_m)$ of the time.

The value $\varphi_m\in(0,1)$ is an input parameter reflecting the hospital's risk aversion for operating over capacity. A high φ_m value means demand will exceed capacity frequently (causing, for example, backlogged demand) whereas a low φ_m value means demand will exceed capacity less frequently (causing, for example, under utilized resources).

Let $\gamma=(t_1,t_2,\ldots,t_N)$ be a vector of chosen times to accept patient types n and let the resulting reward be measured by the discounted weighted sum of completed treatments for decision γ. Discounting future costs by $e^{-\alpha t}$ (where $\alpha\in(0,1)$ is the discount factor) to time 0 ensures that later costs are adequately taken into account. Finding the optimal γ leads to the following optimization problem:

$$\begin{aligned} &\text{maximize} \quad \int_0^T C_t(\gamma)e^{-\alpha t}\,dt \\ &\text{subject to} \quad \mathbb{P}(S_m(t)\geq G_m(t))\leq\varphi_m \quad \forall m,t \end{aligned} \tag{1}$$

where,

$$C_t(\gamma)=\sum_{m=1}^{M}\mathbb{E}[D_m(t)]w_m. \tag{2}$$

Reward function (2) rewards according to the number of treatments completed, and is motivated by the financing structure at the hospital under study. Other choices are possible and the choice can be determined by the underlying decision process. Obvious choices include:

1. $C_t(\gamma)=\sum_{m=1}^{M}\mathbb{P}(S_m(t)\geq G_m(t))$ that rewards according to the fraction of patients that exceed capacity