Computers & Operations Research 43 (2014) 309–317

Contents lists available at ScienceDirect

Computers & Operations Research

journal homepage: www.elsevier.com/locate/caor

Location-arc routing problem: Heuristic approaches and test instances

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ARTICLE INFO

Available online 21 October 2013

Keywords:
Location-routing
Arc routing
Heuristics
Test instances

ABSTRACT

Location-routing is a branch of locational analysis that takes into account distribution aspects. The location-arc routing problem (LARP) considers scenarios where the demand is on the edges rather than being on the nodes of a network (usually a road network is assumed). Examples of such scenarios include locating facilities for postal delivery, garbage collection, road maintenance, winter gritting and street sweeping. This paper presents some heuristic approaches to tackle the LARP, as well as some proposals for benchmark instances (and corresponding results). New constructive and improvement methods are presented and used within different metaheuristic frameworks. Test instances were obtained from the capacitated arc routing problem (CARP) literature and adapted to address the LARP.

1. Introduction

Location-routing problems (LRP) deal with the combination of two types of decisions that often arise: the location of facilities and the design of the distribution routes. While most LRP papers address node routing (see for example [1,2]), one may consider several scenarios where the demand is on the edges rather than being on the nodes of a network (usually a road network is assumed). These problems are referred to in the literature as location-arc routing problems (LARPs), and are derived from the capacitated arc routing problem (CARP) [3].

The LARP is typically overlooked in the literature. It has been shown that node routing problems can be converted into arc routing problems (the capacitated vehicle routing problem – CVRP – can be transformed into the CARP [4]), and that the reverse is also possible, replacing each arc by three [5] or two vertices [6,7], making the two classes of problems equivalent (the same holds true for their location counterparts: the capacitated LRP and the LARP).

Still, for the three transformations of the CARP into the CVRP, the resulting instance requires either fixing of variables or the use of edges with infinite cost. Moreover, the resulting CVRP graph is a complete graph of larger size. Hence, the transformation increases the problem size and the planar structure of a usual CARP graph is lost [8] dramatically changing the number of edges from linear to quadratic. The same can be extrapolated to the LARP, motivating its study using dedicated methods and algorithms.

The first work on the LARP, by Levy and Bodin [9], intended to tackle a practical problem arising in the scheduling of postal carriers in the United States postal service. The developed algorithm used the location–allocation–routing (L–A–R) concept described by [10] for the LRP, which includes three steps: firstly, depots are to be located using a depot selection procedure; secondly, arcs with demand are to be allocated to depots; thirdly, an Euler tour route of minimum traverse cost is determined for each set of arcs allocated to depots.

Ghiani and Laporte [11] addressed an undirected LARP, called the location rural postman problem, in which depots are to be located and routes to be drawn (serving edges with demand), at minimum cost, in an undirected graph. The authors show that the problem can be transformed into a rural postman problem if there is a single depot to open or no bounds on the number of depots. Using an exact branch-and-cut approach they solve the transformed problem.

In subsequent work by Ghiani and Laporte [3] a set of common applications for the LARP is mentioned (mail delivery, garbage collection and road maintenance). Furthermore the authors define the LARP as an extension of one of the three classical arc routing problems: the Chinese postman problem, the rural postman problem, and the CARP. The authors also present some insight into heuristic approaches using the decomposition of the problem into location (L), allocation (A) and routing (R) [10]: location–allocation–routing (L–A–R) and allocation–routing–location (A–R–L).

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http://dx.doi.org/10.1016/j.cor.2013.10.003

Muyldermans [12] presents a variant of the LARP: the p dead-mileage problem. In this problem, unlike the previously addressed LARPs, splitting of the demand is allowed, that is, the client can be serviced more than once. The objective is to minimize dead-mileage (deadheading) and the problem is solved exactly.

Finally, the works by Pia and Filippi [13] and Amaya et al. [14] address variants of the CARP with a structure similar to the LARP, respectively, the CARP with mobile depots and the CARP with refill points. In the first, two different types of vehicles are considered: compactors and satellites. Compactors can be seen as mobile depots for the satellites. The second problem considers servicing of arcs by vehicles that must be refilled at certain nodes (to be determined) in order to complete the service.

From the previously mentioned variants, the LARP addressed here is the one studied by Ghiani and Laporte [11] which can be seen as the arc routing equivalent to the capacitated LRP, and thus an extension to the CARP.

In this paper some heuristic approaches are presented to tackle the LARP, as well as some proposals for benchmark instances (and corresponding results). Regarding the heuristic approaches new constructive and improvement methods are developed and used within different metaheuristic frameworks. The test instances were obtained from the CARP literature and adapted to address the LARP.

The remainder of this paper is outlined as follows. In Section 2 a formal definition of the problem is given. Constructive methods and improvement heuristics are presented in Section 3, and used within different metaheuristic frameworks proposed in Section 4. The developed test instances are addressed in Section 5 as well as the corresponding computational results. Finally, conclusions and future research directions are presented in Section 6.

2. Problem definition

The LARP consists of determining simultaneously depot location and routes in a graph in order to serve a specified set of required arcs under given operational constraints. Muyldermans [12] has shown that, for this problem, an optimal solution exists with the facilities located on the vertices of the graph.

Formally, the LARP can be described on a weighted and directed graph $G=(V, A)$ with vertex set V and set of arcs A. The vertex set V contains a non-empty subset J of m potential depot locations ($J \subseteq V$) with a fixed cost f_j and an associated capacity b_j ($j \in J$). Every arc $a=(i, j)$ in the arc set A has a non-negative traversal cost c_a and a non-negative demand for service d_a. The arcs with positive demand form the subset R of the arcs required to be serviced, only once, by a fleet K of identical vehicles with capacity Q. Vehicles start and end their route in the same depot, and each new vehicle (or route, as it is assumed that each vehicle performs a single route) involves a fixed cost F. The movement from the end i of one required arc to the start j of another required arc without servicing the traversed arcs (either required or not) is known as "deadheading", and has an associated cost denoted by z_{ij} (usually the cost of the shortest path in G from i to j).

The problem aims to determine the set of depots to be opened in J and the tracing of the distribution routes assigned to each open depot in such a way that the sum of fixed and traversal costs to serve all arcs in R is minimized.

Assuming G to be connected, it is possible to transform it into a complete graph $\hat{G}=(\hat{V}, \hat{A})$ where $\hat{V}$ is composed of the set V_R of vertices containing the extremities of the arcs in R ($V_R \subseteq V$), and J ($\hat{V}=V_R \cup J$). As $\hat{G}$ is a complete graph and $V_R \subseteq \hat{V}$, R is a subset of $\hat{A}$. Each arc $a=(i, j)$ in the arc set $\hat{A}$ has a non-negative cost $\hat{c}_a$ which takes on the value c_a if $a \in R$, z_{ij} otherwise.

For any subset S of vertices in $\hat{V}$, let $\delta^+(S)$ ($\delta^-(S)$), be the set of arcs leaving (entering) S, and $L(S)$ the set of arcs with both extremities in S. When S contains a single vertex v, $\delta^+(v)$ is a simplification for $\delta^+(\{v\})$. The following binary variables are used: x_{ak}, equal to 1 if and only if arc $a \in \hat{A}$ is used in the route performed by vehicle $k \in K$; y_j, equal to 1 if and only if depot j is to be opened; and w_{aj}, equal to 1 if and only if the arc $a \in R$ is assigned to depot j. The LARP can be formulated as:

$$\text{(LARP)} \quad \min \quad Z=\sum_{j \in J} f_j y_j + \sum_{a \in \hat{A}} \sum_{k \in K} \hat{c}_a x_{ak} + \sum_{k \in K} \sum_{a \in \delta^+(J)} F x_{ak} \tag{1}$$

$$\text{s.t.}: \quad \sum_{k \in K} x_{ak}=1 \quad \forall a \in R, \tag{2}$$

$$\sum_{a \in R} d_a x_{ak} \le Q \quad \forall k \in K, \tag{3}$$

$$\sum_{a \in \delta^+(i)} x_{ak} - \sum_{a \in \delta^-(i)} x_{ak}=0 \quad \forall i \in \hat{V}, \ \forall k \in K, \tag{4}$$

$$\sum_{a \in \delta^+(J)} x_{ak} \le 1 \quad \forall k \in K, \tag{5}$$

$$\sum_{a \in L(S)} x_{ak} \le |S|-1 \quad \forall k \in K, \ \forall S \subseteq V_R, \tag{6}$$

$$\sum_{b \in \delta^+(j) \cap \delta^-(V_R)} x_{bk} + x_{ak} \le 1 + w_{aj} \quad \forall a \in R, \ \forall j \in J, \ \forall k \in K, \tag{7}$$

$$\sum_{a \in R} d_a w_{aj} \le b_j y_j \quad \forall j \in J, \tag{8}$$

$$x_{ak} \in \{0, 1\} \quad \forall a \in \hat{A}, \ \forall k \in K, \tag{9}$$

$$y_j \in \{0, 1\} \quad \forall j \in J, \tag{10}$$

$$w_{aj} \in \{0, 1\} \quad \forall a \in R, \ \forall j \in J. \tag{11}$$

The objective function (1) minimizes the sum of, respectively, the fixed costs of opening the depots, the costs of all traversed arcs, and the cost of acquiring vehicles. Constraints (2) ensure that each required arc is serviced once by exactly one vehicle. Capacity constraints are satisfied thanks to inequalities (3) and (8). Equalities (4) are the flow conservation constraints which, coupled with constraints (5), ensure the routes return to the departure depot. Constraints (6) are subtour elimination constraints while the set of constraints (7) specify that a required arc must be assigned to a depot in case there is a route linking them. Finally, constraints (9)–(11) define the variables. Note that integrality of w_{aj} can be relaxed to [0, 1] because if not pushed to 1 by (7) minimization will choose for 0 due to (8).

It can be noted that the LARP considered here can be seen as an extension of the CARP, where multiple depots are considered and an additional level of decision is for locating the depots.

3. Constructive methods and improvement heuristics

As the LARP results from the combination of a facility location problem and the CARP, both NP-hard problems [15,4], it is NP-hard. As a consequence, large instances can hardly be solved using exact methods; moreover, sharp bounds on the optimal value are typically hard to obtain. The best way to tackle these problems is then to use heuristic approaches [16] such as constructive methods and improvement heuristics.

Constructive methods are commonly used to obtain initial solutions from which improvement heuristics seek to attain better ones. Furthermore, these approaches are often used as the first step to many metaheuristics. In this section constructive methods (extended augment-merge and extended merge) and improvement heuristics