Computers & Operations Research 38 (2011) 755–770

Contents lists available at ScienceDirect

Computers & Operations Research

journal homepage: www.elsevier.com/locate/caor

Minimizing total completion time on a single machine with a flexible maintenance activity

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ARTICLE INFO

Available online 16 October 2010

Keywords:
Single machine scheduling
Flexible maintenance
Shortest processing time
Dynamic programming
Branch-and-bound

ABSTRACT

A problem of jointly scheduling multiple jobs and a single maintenance activity on a single machine with the objective of minimizing total completion time is considered in this paper. It is assumed that the machine should be stopped for maintenance which takes a constant duration within a predefined period. The problem is generalized from the one with a fixed maintenance in that it relaxes the starting time of the maintenance from a fixed time point to a predefined period. Both resumable and nonresumable cases are studied. First, three properties of an optimal solution to each of the two cases are identified. Then it is shown that the proposed shortest processing time (SPT) algorithm is optimal for the resumable case. As for the nonresumable case, the conditions under which the SPT algorithm is optimal are also specified. Furthermore, it is shown that relaxing the starting time of the maintenance cannot improve the relative error bound of the SPT algorithm. The focus of the paper is presented afterwards, which is to develop a dynamic programming algorithm and a branch-and-bound algorithm to generate an optimal solution for this case. Experimental results show that these algorithms are effective and complementary in dealing with different instances of the problem.

Statement of scope and purpose: In the majority of scheduling problems with preventive maintenance, maintenance periods are assumed to be constant. However, in real industry settings, such periods may be flexible. Therefore, it is necessary to consider scheduling problems with flexible maintenance. This paper focuses on a single machine problem in which job processing and machine maintenance have to be scheduled simultaneously. The objective is to minimize total completion time of jobs for both resumable and nonresumable cases. For the resumable case, a SPT algorithm proposed in this paper is shown to be optimal. On the other hand, for the nonresumable case, the relative worst-case error bound of the SPT algorithm is analyzed, and further, a dynamic programming algorithm and a branch-and-bound algorithm are proposed to solve this problem optimally. Finally, experimental results are provided to show the effectiveness and complementarity of the above algorithms.

1. Introduction

Scheduling problems with preventive maintenance have received increasing attention in the last decade, as the importance of the applications has been recognized. There are two types of preventive maintenance: deterministic maintenance and flexible maintenance. The former means that maintenance periods are determined before jobs are scheduled, i.e., the starting times of the maintenance activities are fixed in advance. Scheduling with this type of maintenance is often referred in literature as “scheduling with availability constraints”, “scheduling with limited machine availability” or “scheduling with fixed non-availability intervals” because during maintenance periods the machine is unavailable for processing any job. The latter means that the schedule of maintenance periods are determined jointly with the schedule of jobs, i.e., the starting times of the maintenance activities are allowed to be flexible and are determined in the scheduling process. This problem is known as “scheduling with flexible maintenance”, “scheduling with unfixed availability constraints”, “integrating preventive maintenance planning and production scheduling”, or “(simultaneously/jointly) scheduling jobs and maintenance activities”.

The first type of maintenance, the deterministic one, has been addressed by significant number of works. Comprehensive reviews were provided by Lee et al. [1], Sanlaville and Schmidt [2], Schmidt [3], and Ma et al. [4]. The following is a review of the works dealing with a subset of the problems, i.e., the single machine total (weighted) completion time problems with only one maintenance activity.

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doi:10.1016/j.cor.2010.09.003

As far as the total completion time of jobs is concerned, while the resumable case can be solved optimally by the shortest processing time (SPT) rule, the corresponding nonresumable case is NP-hard [5] and the SPT rule leads to a relative worst-case error bound of 2/7 [6], where the resumable and nonresumable case are defined by Lee [7]: a job is resumable if it cannot be finished before the unavailable period of a machine and can continue after the machine is available again; on the other hand, it is nonresumable if it has to restart, rather than continue after the machine becomes available.[1] In addition, an approximation algorithm with a tight error bound of 3/17, a parametric $O(n\log n)-$ algorithm with better error bounds and a polynomial time approximation scheme (PTAS) were proposed by Sadfi et al. [8], Breit [9], and He et al. [10], respectively.

When the objective is to minimize the total weighted completion time of jobs, Lee [7] showed that both the resumable and nonresumable cases are NP-hard and the relative error bound of weighted shortest processing time (WSPT) rule can be arbitrarily large even if the weights are proportional to the processing times. For the resumable case, he also developed a combine-algorithm which combines the WSPT and the largest weight rule and showed that its error bound can be arbitrarily large, but when the weights are proportional to the processing times, it leads to a tight error bound of 1. In the paper, a dynamic programming model was also proposed to solve the problem optimally. Furthermore, a heuristic with a tight relative error bound of 1 was proposed by Wang et al. [11] to solve the same problem. In the case that the jobs are nonresumable, Kacem and Chu [12] showed that both WSPT and MWSPT (modified WSPT) have a tight relative error bound of 2 under some conditions. Then, Kacem [13] proposed a 2-approximation algorithm and showed that this bound is tight. Three exact algorithms, a branch-and-bound, a mixed-integer linear programming and a dynamic programming, were proposed by Kacem et al. [14] to solve this problem optimally. Later, new properties and lower bounds were proposed to improve the branch-and-bound algorithm by Kacem and Chu [15].

Besides the literature on the first type of maintenance mentioned above, there are also a few works dealing with the second type of maintenance. Since the maintenance is flexible, most of the works are concerned with nonresumable cases. So, in the subsequent discussions, jobs are assumed to be nonresumable, unless otherwise specified. Based on practical requirements, various kinds of flexible maintenance are considered.

The first kind is that a machine must be maintained after working continuously for a period of time and the maximum allowed continuous working time is fixed. This model were considered by Mosheiov and Sarig [16,17], Lee and Chen [18], Levin et al. [19], Allaoui et al. [20], Qi et al. [21], Qi [22], Sbihi and Varnier [23], and Graves and Lee [24]. In some of those papers, it is assumed that each machine is maintained only once [16–20], while in others, machines must be maintained several times. Meanwhile, the same scheduling model was independently studied by Akturk et al. [25,26] with a different motivation from tool management point of view where maintenance activities are interpreted as tool changes.

The second kind is that each maintenance activity corresponds to a predefined time interval, in which the maintenance activity can be scheduled. Yang et al. [27] were the first to study problem with this kind of maintenance. They considered a single machine makespan problem with only one maintenance activity, demonstrated that the proposed problem is NP-hard and provided some solvable cases and a heuristic. When there are multiple maintenance activities on a single machine, Chen [28] developed four mixed binary integer programming (BIP) models and a heuristic to solve the mean flow time problem. Four problems based on different job characteristics and objective functions are addressed by Lau and Zhang [29]. In the paper, the complexity of these problems and approximation ratio of some proposed heuristics are analyzed. In another paper by Chen [30], both resumable and nonresumable cases were considered for single machine and parallel machine scheduling with the objective of minimizing the total tardiness. To solve them, eight mixed BIP models and a heuristic were proposed. Later, Chen [31] extended his research to the single machine makespan problem, proposed two mixed BIP models and a heuristic, whose worst-case performance bound was showed to be 2 by Xu et al. [32] later. Xu et al. [33] studied a parallel machine scheduling problem where each machine setting is the same as that in Ref. [31]. The objective is to minimize the completion time of the last finished maintenance. They showed that the problem is NP-hard, and unless P=NP, there is no polynomial time ρ—approximation algorithm for this problem for any $\rho<2$. Then they proposed a polynomial time approximation algorithm to solve the problem. When a flexible job shop is considered, Gao et al. [34] proposed a hybrid genetic algorithm to minimize the following three objectives: makespan, maximal machine workload and total workload of the machines.

Other kinds of flexible maintenance were also considered by researchers. For example, Lau and Zhang [29] assumed that a fixed number of jobs must be completed within an available interval; Dell' Amico and Martello [35], Liao et al. [36], and Yang et al. [37] assumed that the machine must be maintained after completing a fixed number of jobs at most; Low et al. [38] assumed two stratagems that a machine should stop to maintain after a periodic time interval or to change tools after completing a fixed number of jobs; Xu et al. [39] assumed that the length of the time interval between any two consecutive maintenance activities is between two given positive numbers in a parallel machine setting, i.e., it belongs to the second kind of maintenance except that the time needed to perform one maintenance activity is increasing linearly instead of being fixed; Cassady and Kutanoglu [40,41], Sortrakul et al. [42], and Ruiz et al. [43] assumed that the time to failure of a machine is governed by a Weibull probability distribution; Berrichi et al. [44], Kaabi et al. [45], Youssef et al. [46], and Chen and Liao [47] defined other different models.

The flexible maintenance model discussed in this paper belongs to the second kind, i.e., during a scheduling period a single machine should be stopped within a predefined time period for maintenance which takes a fixed time to complete. Detailed problem description is given in the next section. To the best knowledge of the authors, the problem has not been dealt with previously with the objective of minimizing the total completion time. This maintenance model is originated from an air-conditioner manufacturer where the second author worked for three years. In this company, there are many types of machines, such as aluminium foil puncher, copper tube bender, copper tube expander, heat exchanger bender, degreasing furnace, drying furnace, synthetic test equipment, fluoride injector, etc. Some simple maintenance of each machine can be done by shop floor workers, such as cleaning, lubricating, fixing screw, and spot inspection. But the more difficult maintenance has to be done by a serviceman from an equipment company affiliated to the same group, such maintenance include: the maintenance of hydraulic system, circuit system and punching program, the replacement of the punching die, the adjustment of expending rod, flanging height and bending angle, and the

[1] Actually, a nonresumable case is the same as a nonpreemptive one, i.e., a job must be completed without interruption once started. Therefore, without loss of generality, nonresumable case and nonpreemptive case are used interchangeably in this paper.