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Evolutionary multiobjective optimization using an outranking-based dominance generalization

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ABSTRACT

One aspect that is often disregarded in the current research on evolutionary multiobjective optimization is the fact that the solution of a multiobjective optimization problem involves not only the search itself, but also a decision making process. Most current approaches concentrate on adapting an evolutionary algorithm to generate the Pareto frontier. In this work, we present a new idea to incorporate preferences into a multi-objective evolutionary algorithm (MOEA). We introduce a binary fuzzy preference relation that expresses the degree of truth of the predicate "***x*** is at least as good as ***y***". On this basis, a strict preference relation with a reasonably high degree of credibility can be established on any population. An alternative ***x*** is not strictly outranked if and only if there does not exist an alternative ***y*** which is strictly preferred to ***x***. It is easy to prove that the best solution is not strictly outranked. For validating our proposed approach, we used the non-dominated sorting genetic algorithm II (NSGA-II), but replacing Pareto dominance by the above non-outranked concept. So, we search for the non-strictly outranked frontier that is a subset of the Pareto frontier. In several instances of a nine-objective knapsack problem our proposal clearly outperforms the standard NSGA-II, achieving non-outranked solutions which are in an obviously privileged zone of the Pareto frontier.

1. Introduction

In real-world optimization problems, the decision-maker (*DM*) is usually concerned with several criteria which determine the quality of solutions. Often, constraints in mathematical programming problems are not actually mandatory; instead, such restrictions are expressing an important desire, a significant *DM* aspiration level about certain system properties. Therefore, most optimization problems can be represented from a multiple objective perspective.

As a consequence of the conflicting nature of the criteria, it is not possible to obtain a single optimum, and, consequently, the ideal solution of a multiobjective problem (MOP) cannot be reached. Hence, to solve a MOP means to find the best compromise solution according to the *DM*'s particular system of preferences (value system). It is easy to prove that the best compromise is a non-dominated solution (i.e., a member of the Pareto optimal set). Most operations research methods for MOPs can be classified into the following categories [1]:

(1) Techniques which perform an *a priori* articulation of the *DM*'s preferences.
(2) Interactive methods, which perform a progressive articulation of the *DM*' preferences.
(3) Generating techniques, which perform an *a posteriori* articulation of the *DM*'s preferences (search before making decisions).

Since David Schaffer's seminal work (cf. [2]), multi-objective evolutionary algorithms (MOEAs) have become a very popular search engine for solving multiobjective programming problems. MOEAs are very attractive to solve MOPs because they deal simultaneously with a set of possible solutions (the MOEA's population) which allows them to obtain an approximation of the Pareto frontier in a single algorithm's run. Thus, by using MOEAs the *DM* and/or the decision analyst does not need to perform a set of separate single-objective optimizations as normally required when using operations research methods. Additionally, MOEAs are more robust regarding the shape or continuity of the Pareto front, whereas these two issues are a real concern for classical optimization methods (cf. [3]). However, one aspect that is often disregarded in the MOEAs

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literature is the fact that the solution of a problem involves not only the search process, but also (and normally, more important) the decision making process. Most current approaches in the evolutionary multiobjective optimization literature concentrate on adapting an evolutionary algorithm to generate an approximation of the Pareto optimal set. Nevertheless, finding this set does not solve the problem. The *DM* still has to choose the best compromise solution out of that set. This is not a very difficult task when dealing with problems having 2 or 3 objectives. However, as the number of criteria increases, two important difficulties arise:

(a) The algorithm's capacity to find this Pareto frontier quickly degrades.
(b) It becomes harder, or even impossible for the *DM* to establish valid judgments in order to compare solutions with several conflicting criteria.

Here, we propose a combined approach, with an a priori articulation of preferences followed by a generating process of a specific (i.e., desirable) zone of the Pareto frontier. Using a fuzzy outranking relation, a strict preference relation in the sense of [4] can be established in any population. Our proposal is based on finding a subset of the Pareto frontier composed of solutions for which no other solutions exist which are preferred to the first ones. This non-outranked concept will be used instead of dominance when performing the evolutionary search.

The remainder of this paper is organized as follows. An outranking model of multicriteria preferences is outlined in Section 2, and on this basis the proposed dominance generalization is detailed in Section 3. Our algorithmic proposal is discussed in Section 4 and illustrated by some computer experiments in Section 5. Finally, we draw brief concluding remarks in Section 6.

2. An outranking model of preferences

Let G be the set of objective functions of a multicriteria optimization problem and O its objective space. An element $\boldsymbol{x} \in O$ is a vector $(x_1, \ldots, x_n)$, where x_i is the i-th objective value. Let us suppose that for each criterion j there is a relational system of preferences (P_j, I_j) (preference, indifference) which is complete on the domain of the j-th criterion (G_j). That is, $\forall (x_j, y_j) \in G_j \times G_j$ one and only one of the following statements is true:

– $x_j P_j y_j$

– $y_j P_j x_j$

– $x_j I_j y_j$ (1)

Formulation (1) allows indifference thresholds in order to model some kind of imprecise one-dimensional preferences. It should be noticed that the relational system of preferences given by (1) is more general than the usual formulations which consider only true criteria (that is, $x_j \neq y_j$ implies non-indifference). Without loss of generality, in the following we suppose that $x_j P_j y_j \Rightarrow x_j > y_j$.

Let us establish the following central premise: for each $(\boldsymbol{x},\boldsymbol{y}) \in O \times O$, the *DM* and the decision analyst (working together) are able to create a fuzzy predicate modeling the degree of truth of the statement "$\boldsymbol{x}$ is at least as good as $\boldsymbol{y}$ from the *DM*'s point of view".

Amongst different ways to create that predicate, we shall describe below an outranking approach based on ELECTRE methods:

A proposition $\boldsymbol{x}S\boldsymbol{y}$ ("$\boldsymbol{x}$ outranks $\boldsymbol{y}$") ("$\boldsymbol{x}$ seems at least as good as $\boldsymbol{y}$") holds if and only if the coalition of criteria in agreement with this proposition is strong enough and there is no important coalition discordant with it (cf. [5]). It can be expressed by the following logical

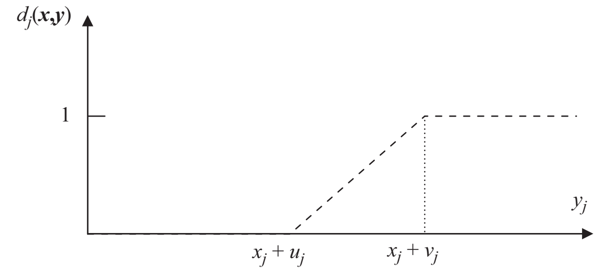

Fig. 1. Partial discordance relation $d_j(\boldsymbol{x},\boldsymbol{y})$.

equivalence (cf. [6]):

$$\boldsymbol{x}S\boldsymbol{y} \Leftrightarrow C(\boldsymbol{x},\boldsymbol{y}) \wedge \sim D(\boldsymbol{x},\boldsymbol{y}) \quad (2)$$

where $C(\boldsymbol{x},\boldsymbol{y}$ is the predicate about the strength of the concordance coalition; $D(\boldsymbol{x},\boldsymbol{y})$ the predicate about the strength of the discordance coalition; $\wedge$ and $\sim$ are logical connectives for conjunction and negation, respectively.

Let $c(\boldsymbol{x},\boldsymbol{y})$ and $d(\boldsymbol{x},\boldsymbol{y})$ denote the degree of truth of the predicates $C(\boldsymbol{x},\boldsymbol{y})$ and $D(\boldsymbol{x},\boldsymbol{y})$. From (2), the degree of truth of $\boldsymbol{x}S\boldsymbol{y}$ can be calculated as in the ELECTRE-III method:

$$\sigma(\boldsymbol{x},\boldsymbol{y}) = c(\boldsymbol{x},\boldsymbol{y}) \cdot N(d(\boldsymbol{x},\boldsymbol{y})) \quad (3)$$

where $N(d(\boldsymbol{x},\boldsymbol{y}))$ denotes the degree of truth of the non-discordance predicate.

As in the earlier versions of the ELECTRE methods, we shall take

$$c(\boldsymbol{x},\boldsymbol{y}) = \sum_{j \in C_{x,y}} w_j \quad (4)$$

where $C_{x,y} = \{j \in G \text{ such that } x_j P_j y_j \vee x_j I_j y_j\}$; w's denote "weights" $(w_1 + w_2 + \cdots + w_n = 1)$ and $\vee$ the symbol for disjunction.

Let $D_{x,y} = \{j \in G \text{ such that } y_j P_j x_j\}$ be the discordance coalition with $\boldsymbol{x}S\boldsymbol{y}$. The intensity of discordance is measured in comparison with a veto threshold v_j, which is the maximum difference $y_j - x_j$ compatible with $\sigma(\boldsymbol{x},\boldsymbol{y}) > 0$. Following Mousseau and Dias [7], we shall use here a simplification of the original formulation of the discordance indices in the ELECTRE-III method which is given by

$$N(d(\boldsymbol{x},\boldsymbol{y})) = \min_{j \in D_{x,y}} [1 - d_j(\boldsymbol{x},\boldsymbol{y})] \quad (5)$$

$$d_j(\boldsymbol{x},\boldsymbol{y}) = \begin{cases} 1 & \text{iff } \nabla_j \geq v_j \\ (\nabla_j - u_j)/(v_j - u_j) & \text{iff } u_j < \nabla_j < v_j \\ 0 & \text{iff } \nabla_j \leq u_j \end{cases} \quad (6)$$

where $\nabla_j = y_j - x_j$ and u_j is a discordance threshold (see Fig. 1).

In practical situations the decision-maker supported by a potential decision-analyst should assess the set of model's parameters which are needed to evaluate σ. This is not an easy task, since decision-makers usually have difficulties in specifying outranking parameters and require an intense support by a decision analyst. To facilitate this process, the pair *DM*-decision analyst can use the preference disaggregation-analysis (*PDA*) paradigm (cf. [8]), which has received an increasing interest from the multicriteria decision support community. *PDA* infers the model's parameters from holistic judgments provided by the *DM*. Those judgments may be obtained from different sources (past decisions, decisions made for a limited set of fictitious objects (actions), or decisions taken for a subset of the objects under consideration for which the *DM* can easily make a judgment [9]). In the framework of outranking methods *PDA* has been recently approached by [10–12].