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Scheduling of coupled tasks and one-machine no-wait robotic cells ☆

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Abstract

Coupled task scheduling problems have been known for more than 25 years. Several complexity results have been established in the meantime, but the status of the identical task case remains still unsettled. We describe a new class of equivalent one-machine no-wait robotic cell problems. It turns out that scheduling of identical coupled tasks corresponds to the production of a single part type in the robotic cell. We shall describe new algorithmic procedures to solve this robotic cell problem, allowing lower and upper bounds on the production time and discussing in particular cyclic production plans.

Keywords: Coupled tasks; Robotic cells; Cyclic production

1. Coupled tasks

A set of n coupled tasks are to be scheduled on a single machine. A coupled task j consists of two operations, which have processing times a_j and b_j, to be executed on the machine. These two operations have to be executed in a specified order and have a separation time of exactly L_j time units (the time between the completion of the first operation and the start of the second). The objective is to minimize the makespan $C_{\max}$ or, in the case of a cyclic schedule, to maximize the throughput rate. Let us denote this problem as Problem (C).

The coupled-task problem was first studied by [1] in the context of scheduling operations for a radar, while [2] presents a more detailed case study involving a multifunction radar system. For radar scheduling applications, the first task is a pulse transmission and the second task is a pulse reception. The separation between tasks is the time for a pulse of energy to be transmitted to a potential target and then be reflected back to the radar. The radar system studied by Orman et al. [2] is used in a military environment for weapon guidance, tracking targets, and surveying volumes of space to find targets.

In [3] the autor describe several other potential applications of the model. In a chemical plant where the same processor is used to perform several operations on the same job, a specified amount of time must elapse between

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Table 1
Table of complexities

Strongly NP-hard	$a_j;\ L_j;\ b_j$
	$a_j = L_j = b_j$
	$a_j = a;\ L_j;\ b_j = b$
	$a_j = a;\ L_j = L;\ b_j$
Open	$a_j = a;\ L_j = L;\ b_j = b$
Polynomial	$a_j = L_j = p;\ b_j$
	$a_j = b_j = p;\ L_j = L$

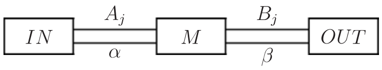

Fig. 1. One-machine robotic cell.

operations due to the chemical reactions involved. A further application occurs for the scheduling of a workstation within a flexible manufacturing system. The workstation can perform different types of operations, although refixturing of the part is needed between successive operations. This refixturing is performed at a load/unload station, and while this is undertaken the workstation can be used to process other jobs.

There are various results concerned with this problem [4], but the complexity for identical tasks ($a_j = a,\ \ L_j = L,\ b_j = b$) is still open (see Table 1 from [4]). The best known algorithm for n identical tasks, to our knowledge, has complexity $O(nr^{2L})$, where $r \leqslant \sqrt[a-1]{a}$ [5]. Note, however, that this is not an algorithm that is polynomial in the size of the instance. This would be polynomial in $\log \max(n, a, L, b)$ or just in $\log \max(a, L, b)$ in the cyclic case.

In the subsequent sections of the paper, we describe a robotic cell problem which is equivalent to the coupled-task problem (Sections 2 and 3). In Section 4, we report numerical results for problems where the separation times have a certain tolerance. Section 5 treats the case of a cyclic production.

2. One-machine no-wait robotic cell

We consider a robotic cell composed of an input station (*IN*), an output station (*OUT*), and a machine (*M*). *IN* and *OUT* have infinite capacity to store the raw material for the parts to be produced and the finished parts, respectively. The machine *M* can treat any number of parts simultaneously (for instance in a chemical tank). Note that this is an assumption that usually is not made in the robotic cell literature. This arrangement is shown in Fig. 1.

A single transporter, a robot, carries out the material handling between *IN*, *M* and *OUT*. It has unit capacity (it can carry one part at a time), but it can bring a part to *M*, leave it there and pick up a finished part that is transported to *OUT*.

A task T_j consists of the following operations: the corresponding material is to be transported from *IN* to *M* in A_j time units that may in the general case depend on *j*. The part is then processed for p_j time units at *M* and after being processed, has to be transferred without delay (no-wait case) in B_j time units to *OUT*. For the complete process, we also consider the movements of the empty robot from *M* to *IN* (α time units), from *OUT* to *M* (β time units) and from *OUT* to *IN* (γ time units). The objective is to execute n tasks with minimal makespan $C_{\max}$, or in the cyclic case, to maximize the throughput rate (for instance for given proportions for the different part types).

Different classes of this problem may be defined by imposing various conditions on the travel times of the empty robot. We distinguish the following cases:

(R_a) additive problem: $\gamma = \alpha + \beta$,
(R_e) equidistant problem: $\alpha = \beta$ and $\gamma < \alpha + \beta$,
(R_g) general problem with arbitrary α, β, γ.