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Scheduling jobs on a single machine to maximize the total revenue of jobs

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Abstract

In today's hyper-competitive marketplace, many products like memory chips and computers are characterized by short life cycles and rapidly declining sales prices. This implies that the amount of revenue generated as a result of completing a product (job) may be decreasing as its completion time is delayed. In such an environment, there is a decided preference for the maximization of product revenues as an important objective. Based on the assumption that the decreasing rate of revenue is dependent on their product types, we intend to develop a searching algorithm and some heuristic algorithms to locate optimal and near-optimal job sequences, respectively, and thereby maximize total earned revenue.

Keywords: Scheduling; Single machine; Total revenue; Branch-and-bound algorithm

1. Introduction

This paper proposes to explore a single machine scheduling problem, where each job in a set $N = \{1, 2, \ldots, n\}$ is to be processed consecutively. Job i, $i \in N$, is specified by a processing time p_i, an original revenue (profit) $R_i > 0$ and a decreasing rate $\alpha_i \geqslant 0$. When job i is completed at time C_i and is then delivered to the buyer or is sold in the marketplace, it is assumed to generate an amount of revenue (or a profit) equivalent to $R_i e^{-\alpha_i C_i}$. This says the revenue generated by a job is decreasing in the form of an exponential function of its completion time. The jobs can represent batches composed of many similar product types or time-consuming tasks, so they generally require days or even weeks to complete. Under such a scenario, the reward received upon the completion of a job may change over time due to the marketplace or customer needs. Particularly in today's hyper-competitive marketplace, many products, like DVD-R, memory chips and computers, are characterized by short life cycles and rapidly declining sales prices. As can be seen from Fig. 1, the prices relating to three types of DVD-R decline with different speeds, and their trends approximate to the exponential model assumed in this study. As a result, the job-dependent rate α_i in essence is a combination of the risk-less discounted rates of net present value (NPV) in project scheduling and the declining rates of product prices. With the job-dependent rate α_i, our concern is to sequence jobs in order to maximize their total revenue.

In literature, many scheduling problems take into consideration job weights, which imply holding or inventory costs per unit. Let w_i and C_i denote the weight and completion time of job i, respectively. The total weighted completion

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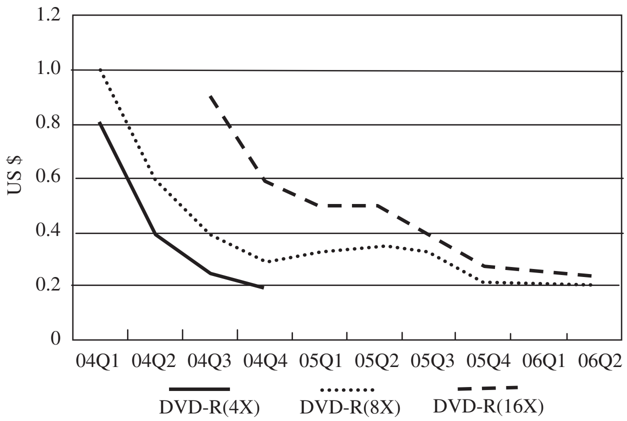

Fig. 1. A plot for the prices of DVD-R.

time of all jobs in set N can be formulated by

$$\sum_{i=1}^{n} w_i C_i.$$

Suppose the ith completed job has weight $w_{(i)}$ and processing time $p_{(i)}$, where $i = 1, 2, \ldots, n$. Then one of the optimal sequences with a minimal total weighted completion time on a single machine is a sequence of jobs arranged so that

$$\frac{w_{(1)}}{p_{(1)}} \geqslant \frac{w_2}{p_2} \geqslant \cdots \geqslant \frac{w_{(n)}}{p_{(n)}},$$

which is the well-known "*ratio rule*" of Smith [1] or *weighted shortest processing time* (WSPT) rule. The special case of $w_i = 1$ for $i = 1, 2, \ldots, n$ gives rise to the *shortest processing time* (SPT) rule. Since Smith's work, many distinctive studies have discussed solution methods for the total weighted completion time problems with job release dates and/or job due dates, for which Posner [2] demonstrated their reducibility.

Extending from the minimization of total weighted completion time, Rothkopf [3] considered the time value of money and referred to the discounted total weighted completion time as

$$\sum w_i (1 - \mathrm{e}^{-\alpha C_i}),$$

where α (> 0) is a continuous discount factor that is common to all jobs. The discounted total weighted completion time can be minimized by the *weighted discounted SPT* (WDSPT) rule, with jobs being scheduled in non-increasing order of the ratio as follows:

$$w_i \mathrm{e}^{-\alpha p_i} / (1 - \mathrm{e}^{-\alpha p_i}).$$

In addition, Lawler and Sivazlian [4] generalized Smith's ratio rule to solve the problems that involved discounted linear delay costs, discounted linear processing costs, discounted resetting and processing costs, and linear combinations of these costs. Rothkopf and Smith [5] proved that, except where the cost objectives are a linear function or an exponential function with a *job-independent* discount rate α, no other single machine scheduling problems with a cost function relating to the completion times of jobs can be solved by means of a simple priority index rule, i.e., polynomially solvable.

A quadratic function of the completion times of jobs was discussed by Townsend [6]. He first established some ordering criteria to arrange a pair of "adjacent" jobs in a sequence, and then incorporated them into a branch-and-bound algorithm. By imposing different constraints on job weights, Merten and Muller [7], Kanet [8] and Woeginger [9] paid attention to the minimization problem of the weighted variance of job completion times. The notion of variance mainly measures the deviations between job completion times and average completion time. Alidaee [10,11] were concerned about a non-linear non-decreasing cost function depending on the completion times of jobs. He designed some heuristic