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Setting due dates in a stochastic single machine environment

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Abstract

A set of n jobs with statistically independent random processing times has to be processed on a single machine without idling between jobs and without preemption. It is required to set due dates and promise them to customers. During the production stage, earliness and tardiness against the promised due dates will be penalized. The goal is to minimize the total expected penalties. We consider two due date setting procedures with optimum customer service level, and an $O(n \log n)$ time complexity. We show that one is asymptotically optimal but the other is not. Both heuristics include safety time and the sequence remains the same regardless of disruptions, so the result is robust. For the normal distribution we provide sufficient optimality conditions, precedence relationships that the optimal sequence must obey, and tight bounds.

Keywords: Stochastic scheduling; Due-dates; Customer service; Robust scheduling

1. Introduction

Setting due dates (or delivery dates) involves communications with the customer. They are either promised or negotiated. To support negotiation or to determine what the delivery date should be, the supplier's representative always requires a scheduling- or a capacity-booking mechanism. By scheduling the customer's order against current portfolio of orders she obtains a realistic delivery date, and then promises it to the customer. This makes the research on assigning due dates much more useful in practice then the research on (given) due-date scheduling [1,2].

During the production stage, both earliness and tardiness against the promised due date is penalized [3]. The tardiness penalty models a natural part of business relations. Frequently it is approximately

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proportional to the delay. The penalty for being early is less straightforward. Earliness cost mostly reflects *internal* expenses, e.g., inventory holding cost, or loss due to late payment (the product is ready, but the customer pays on the due date). The natural business criterion for setting due dates is the expected sum of earliness–tardiness penalties. For example, ports have to set dates for processing ships at big cargo terminals and, once scheduled, it is important to avoid delaying ships or idling the port facilities. Until the scheduled time, however, the port has no responsibility for the ships' time, and indeed ships can resort to various schemes to utilize scheduled delays, if any, e.g., by scheduling visits to other ports or by saving costs by selecting a more economical speed.

The problem is inherently connected to the stochastic nature of processing times. First, once we consider stochastic variation, assuming continuous processing time distributions, every job will almost surely be either early or tardy. Second, the earliness–tardiness of any job depends not only on its own variability, but also on its position in the sequence. A highly variable job placed at the beginning of the sequence will add to variability of all the jobs placed after it. Third, the delivery dates should be defined not only by the expected completion of the job, but also with respect to customer service level.

Since earliness is typically not a concern for customers—they don't even have to know about it—service levels may be computed as the probability of avoiding tardiness. Note that if we set the due date as an expected completion time of a job, then the service level for any symmetrical distribution will be 50%. Nonetheless, the optimal service level is associated with the relative magnitude of the earliness and tardiness costs, so it should often be much higher than 50% [4].

Stochastic models mostly assume given deterministic due dates and stochastic processing times. An example of this approach is Soroush and Fredendall [5]. However, the option that is relevant to practice involves setting deterministic due-dates under assumption of stochastic processing times. Unfortunately, not much had been published in this area. Probably, the first to attract attention to this important problem was Cheng [6]. His model combines sequencing and due-date setting and he achieves optimal results for some special cases. Soroush [7] extended the results of [5] to the case where due dates are part of the problem. He compared the results of two heuristics and found one of them—to which we refer as the primary heuristic—often superior, especially in examples with more jobs. Other due-dates environments have also been investigated, for example, a stochastic counterpart of the well-known earliness–tardiness scheduling problem with a common due date, in which n stochastic jobs are to be processed on a single machine [8]. The objective was to minimize the expectation of a weighted combination of the earliness penalty, the tardiness penalty, and the flow-time penalty. One of the main results was that an optimal sequence for the problem must be V-shaped with respect to the mean processing times. Two algorithms were proposed, which can generate optimal or near-optimal solutions in pseudo-polynomial time. In our paper we investigate further the problem studied by Soroush [7]. We show that the heuristic that seemed to perform better is asymptotically optimal, while the other is not. We also provide bounds and conditions that make possible the optimal solution of the problem by methods such as branch and bound.

In the next section, we formulate the problem. In Section 3, we consider the primary $O(n \log n)$ heuristic. In Section 4, we derive a basic lower bound that this approximation provides. In Section 5, we show that under mild conditions the primary heuristic is asymptotically optimal, but the other heuristic is not. In Section 6, we provide a sufficient condition for optimality. In Section 7, we present some precedence relationships that might be useful in searching for an optimal solution. In Section 8, we develop a tighter lower bound to support solution by branch and bound. Section 9 is the conclusion.