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A pricing scheme for combinatorial auctions based on bundle sizes

Dirk Briskorn [a,*], Kurt Jørnsten [b], Philipp Zeise [a]

[a] Bergische Universität Wuppertal, Schumpeter School of Business and Economics, Professur für BWL, insbesondere Produktion und Logistik, Rainer-Gruenter-Str. 21, 42119 Wuppertal, Germany

[b] Norwegian School of Economics and Business Administration, Department of Finance and Management Science, Bergen, Norway

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ABSTRACT

In combinatorial auctions not only single items but also bundles of items are sold simultaneously. A substantial ingredient to an auction mechanism is the way prices of bundles are determined. Prices determine the auctioneer's revenue and, ideally, justify the outcome of the auction to the bidder. Each bidder should be able to see why he won or lost a certain bundle comparing the determined price for a bundle and his bid's value. It is well known that linear prices cannot guarantee such a justification. We propose a new pricing scheme adding prices for bundle sizes to the traditional linear prices for items. We analyze this scheme and evaluate its ability to provide prices supporting a given allocation by means of a computational study using a well established combinatorial auctions test suite. We also compare our scheme to a scheme from literature with respect to the ability to generate market clearing prices.

1. Introduction

Auctions in which bidders are allowed to bid on bundles of items and the bidder gets either each item in the bundle if the bid wins or no item at all if the bid looses are called combinatorial auctions (CA). This auction type has been widely addressed in the scientific literature (see, [14] for a survey).

Obviously, CAs are much more complex than auctions where only one item is involved. Several decisions have to be made carefully when establishing a CA. For example, the auction format, that is the pieces of information given to the auctioneer when bidding and whether there is only a single round of bidding or there will be multiple rounds, must be determined. Furthermore, it must be decided (i) how the winning bids are determined, (ii) how much the winning bidders are charged, and (iii) what kind of information is given to the bidders after the winning bids have been determined. All of these decisions have been treated in the literature (see, e.g. [11,14,31]). In an iterative CA an allocation is determined in each of multiple rounds and feedback (e.g. prices) is given to the bidders who may adjust their bids in the next round accordingly. Here, prices are particularly important for the auction to converge to an efficient allocation.

In the paper at hand we propose a scheme to determine prices that support the allocation. On basis of these prices an explanation of winning and losing bids can be given to bidders that are charged accordingly. Thus, the pricing scheme is part of decisions (ii) and (iii).

When determining prices for bundles the most important requirements are that, first, for winning bids the corresponding bundle price does not exceed the bid's value and, second, for losing bids the corresponding bundle price does not undershoot the bid's value. Both are substantial since winning bidders will not agree to pay more than the bid's value and losing bidders will complain if the bundle price is less what they are willing to pay. We say that bundle prices support the allocation if they fulfill both requirements. A stronger concept known from literature is the one of competitive equilibrium prices (see, e.g. [31]).

The easiest price system that can be used is a linear price system. Here, for each item an individual price is determined and bundle prices are derived as the sum of the prices of the items contained in the bundle. A set of item prices constitutes a market clearing price system if they balance out supply and demand. That is, the induced bundle prices support the given allocation and each item is sold or its price equals zero. Apart from being easy to understand the linear price system also has the important property of being anonymous. However, it is well known that linear prices cannot guarantee bundle prices that support a winning allocation. Accordingly, approximations in form of pseudo dual linear prices have been developed e.g. in Drexl et al. [16] and have been used in many CA settings by among others [17,20,32]. How

* Corresponding author. Tel. +49 202 439 1104.
E-mail addresses: briskorn@uni-wuppertal.de (D. Briskorn), kurt.jornsten@nhh.no (K. Jørnsten), zeise@uni-wuppertal.de (P. Zeise).
URL: http://www.prodlog.uni-wuppertal.de/ (D. Briskorn).

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linear item prices and/or pseudo dual prices work in a CA setting has been investigated experimentally by several authors, e.g. Bichler et al. [4,5], Dunford et al. [17], and Scheffel et al. [34]. However, as shown by Bikhchandani and Ostroy [6,7] and de Vries and Vohra [14], in order to guarantee supporting prices one has to use at least non-linear anonymous prices. Problems with such prices are that they are less intuitive for agents and also not as easy to communicate.

We restrict ourselves to the OR bidding language where each bidder can win multiple bundles. Arguably, the OR bidding language has its limitations when items are substitutes. However, as pointed out by Bikhchandani and Ostroy [6] and de Vries and Vohra [14], bidder-dependent prices may be necessary for supporting allocations in auctions allowing XOR bids. However, there are non-linear anonymous pricing schemes supporting allocations in auctions limited to OR bids. For example, it is sufficient to set the price for a bundle to the highest value of a bid on this bundle. Despite of its limitations the OR bidding language has shown its merits not only in laboratory but also in real world applications, see Cantillon and Pesendorfer [9] and Ledyard et al. [23]. In a pre-auction phase like the one described in Cantillon and Pesendorfer [9], bidders can be checked for sufficient operational capacity to handle multiple winning bids. Martin et al. [27] and Meeus et al. [28] describe auctions for electricity markets with bids that are similar to OR bids in that each bidder can win an arbitrary number of bids or each bidder has only one bid ("single-minded bidders"). In general, OR bids are sufficient to express bidders' valuations if bidders are single-minded. Ledyard [22] reports that this is not unrealistic and refers to broadband auctions as an example. Therefore, we use the OR bidding language as a test bed for the anonymous scheme under consideration. Note that using dummy bids (one dummy bid for each bidder) we can simulate the XOR language. In this case, the same pricing scheme can be applied to XOR bids then. However, this comes at the cost of prices being bidder-dependent.

The paper at hand contributes to this field by proposing a new anonymous pricing format extending the linear price system by using a non-linear two-part tariff pricing system. The focus, here, is on anonymity and simplicity using a format which is well known and can be easily understood by bidders. We discuss several variants of the new format and how they compare to each other with regard to the ability of providing supporting prices for a given allocation. In particular, we do not address any game-theoretic properties of iterative auctions when the new pricing formats are employed. Finally, we give empirical evidence for the potential of the new format by means of a computational study based on the combinatorial auction test suite (CATS) provided in Leyton-Brown [25] and discussed in Leyton-Brown and Shoham [26].

The paper is organized as follows. In Section 2 winner determination and pricing in CAs is reviewed. Section 3 proposes the new format which takes bundle sizes into account and Section 4 provides a theoretical analysis and an empirical evaluation. Finally, Section 5 concludes the paper.

2. Winner determination and pricing

In a (single round) CA we are confronted by the following setting. We have given a set I of items to be sold and a set J of bids where each bid $j \in J$ is defined by a subset I_j of items and a value v_j. Each bid j expresses the willingness of a bidder to pay v_j monetary units in order to obtain the bundle I_j of items. Now, the task of the auctioneer is to identify a set of winning bids meaning that each winning bidder gets the items addressed by the bid. Of course, the set of winning bids must be non-overlapping in items. By far the most common objective when identifying the set of winning bids is to maximize total value of winning bids. This problem, namely the winner determination problem (WDP) with OR bids can be represented as an integer program (IP) as follows. Here, binary parameter $a_{i,j}$ equals 1 if and only if $i \in I_j$ and binary variable x_j equals 1 if and only if bid j wins.

$$\text{maximize} \quad \sum_{j \in J} v_j x_j \tag{1}$$

$$\text{s.t.} \quad \sum_{j \in J} a_{i,j} x_j \le 1 \quad \forall i \in I \tag{2}$$

$$x_j \in \{0,1\} \quad \forall j \in J \tag{3}$$

It can be seen easily that (1)–(3) represent the objective of maximizing winning bids' total value, the requirement of non-overlapping winning bids, and the variables' domains, respectively.

The WDP is an NP-hard combinatorial optimization problem (even if each bid does not involve more than three items, that is $|I_j| \le 3$ for each $j \in J$) and has been addressed in a vast amount of papers, see Lehmann et al. [24], Müller [29], and Sandholm [33] for an overview of problem settings, computational complexity, and exact solution approaches. However, solving the WDP is not the focus of the paper at hand. Accordingly, we assume that we have a tool available that solves it sufficiently fast. The solution of the WDP then can be represented by the set $J^1, J^1 \subseteq J$, of winning bids. Let $J^0, J^0 = J \setminus J^1$, be the set of losing bids.

Now, when choosing a pricing format, that is a mechanism to derive bundle prices, several requirements should be taken into account:

1. The prices should be considered non-discriminating and, therefore, be anonymous.
2. Prices should support the given allocation. This serves as a mean to justify the outcome. Moreover, in case of iterative auctions it enables the bidders to adjust their bids in the following rounds.
3. It should be easy enough for the bidders to understand and also have a good economic interpretation.

A prominent example for a pricing format is to set linear prices for items. Linear prices are intuitive and, therefore, provide insights why a certain bid won/lost. In particular, linear prices also provide prices for bundles for which no bid has been given. This is important especially in CAs with multiple rounds.

Considering the linear programming relaxation of the IP representing WDP we get a dual price for each constraint, i.e. a dual price for each item where

1. item prices are non-negative,
2. item prices for items not being sold equal zero,
3. total item price in bids is at least equal to the bid's value, and
4. total item price in winning bids, that is bids with $x_j > 0$, equals the bid's value.

However, if no optimum solution to the linear programming relaxation of the IP is integer, no linear prices with the desired properties exist. There have been several attempts to come up with pseudo-dual prices that is prices having two of the above-mentioned properties while minimizing violation of the third one (according to various formal objectives), see Drexl and Jørnsten [15], Drexl et al. [16], Hoffman [19], and Rassenti et al. [32]. The pricing scheme proposed in the paper at hand can be seen to employ the linear item prices as a base to which a second component is added. Doing so, we obtain a pricing scheme which is potentially more expressive if it comes to supporting allocations.