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The Steiner traveling salesman problem with online advanced edge blockages

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ABSTRACT

The package delivery in an urban road network is formulated as an online Steiner traveling salesman problem, where the driver (*i.e.* the salesman) receives road (*i.e.* edge) blockage messages when he is at a certain distance to the respective blocked edges. Such road blockages are referred to as *advanced* information. With these online advanced road blockages, the driver wishes to deliver all the packages to their respective customers and returns back to the service depot through a shortest route. During the entire delivery process, there will be at most k road blockages, and they are non-recoverable. When the driver knows about road blockages at a distance αOPT, where $\alpha \in [0, 1]$ is referred to as the *forecasting ratio* and OPT denotes the length of the offline shortest route, we first prove that $\max\{(1-2\alpha)k+1, 1\}$ is a lower bound on the competitive ratio. We then present a polynomial time online algorithm with a competitive ratio very close to this lower bound. Computational results show that our algorithm is efficient and produces near optimal solutions. Similar results for a variation, in which the driver does not need to return to the service depot, are also achieved.

1. Introduction

In the past decade, we have all witnessed the rapid growth of *e*-business world wide. Especially in the last few years in China, the *business-to-customer* (B2C) companies, Amazon (www.amazon.cn), Jingdong (www.jd.com) and Tianmao (www.tmall.com) to name a few, have been competing fiercely. Besides the availability of unimaginable variety of products, the key factor leading to their huge success is the instant delivery. Typically, Yamaxun (or Amazon in English) explicitly gives out the package arrival date at the time an order is made; to be even more competitive, Jingdong promises a half-day delivery time in more than 50 big cities in China, and a *3-h delivery* service in super cities like Beijing and Shanghai. As a result, there are thousands of trucks carrying hundreds of thousands of packages driving on the urban road networks any time during the day.

The huge number of service vehicles, together with even more private vehicles, have caused constant traffic congestion in most of the large cities in China. The package delivery truck drivers, playing an important role in this fast delivery *e*-business, have to enable themselves in finding a short route facing various kinds of sudden road blockages. Fortunately, with the assistance of the intelligent traffic system and the wildly use of smart mobile devices, most road blockages can be broadcasted to the drivers within the vicinity or even be forecasted to the drivers within the vicinity with a very high accuracy. In the literature, Min and Wynter [24] were able to predict the average traffic speed for each 5-min interval 1 h in advance; Daraghmi et al. [8] have predicted the traffic speed 20 min in advance during the peak hours with only about 10% mean absolute percentage error.

The package delivery can be readily modeled as a *Steiner traveling salesman problem* (sTSP) [7], in which we are given an edge-weighted undirected graph $G = (V, E)$ modeling the road network, the weight $w(e)$ of an edge e represents the travel time by the salesman or the delivery truck driver, and there is a subset D of destination vertices each represents a customer. It is important to note that the travel time between two vertices is determined by the physical distance between the two vertices and the road condition; *in this paper, we use edge weight, travel time, and distance interchangeably* (here, distance is normalized over the road condition, thus to represent the travel time). The salesman starts from the service depot, needs to visit every destination at least once to deliver the respective package(s), and lastly returns back

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to the service depot. The optimization goal is to find a shortest route (shortest in the sense of both travel time and *normalized* distance). We assume without loss of generality that the edge weight function satisfies the triangle inequality. Note that the sTSP is a generalization of the classic *traveling salesman problem* (TSP) [12], in which the graph G is a complete graph and every vertex of V is a destination (*i.e.,* $D=V$) to be visited exactly once.

The package delivery with online advanced road blockages can thus be modeled as the sTSP with *online advanced edge blockages*. Here "advanced" means that an edge blockage is known to the salesman when he is at a certain distance (or travel time) to the blocked edge. In other words, if the salesman were to reach the involved edge via the shortest path from his current position in the current graph, then at the time he arrives the edge would be blocked. The "online" means these advanced edge blockages are radioed to the salesman in an online fashion. We assume that during the entire delivery process, there will be *at most* k edge blockages; because the entire delivery process is relatively short while the edge blockages can last much longer, the edge blockages are *non-recoverable* to the salesman. Let OPT denote the length (or the total travel time) of the shortest offline route. The forecasting ratio α, $0 \le \alpha \le 1$, specifies that the salesman knows about each edge blockage when he is at distance αOPT to the edge. When $\alpha=0$, the online edge blockages are said *real-time* [35].

1.1. Literature review

There is a vast amount of literature on the TSP. The interested readers may refer to the book by Lawler et al. [20], and for more recent results to [31]. In summary, the TSP is NP-hard and it does not admit any constant factor approximation [12]. For the relevant case where the edge weights satisfying the triangle inequality, the TSP can be approximated within 1.5 [5].

The Steiner TSP, denoted as sTSP, a special case of the general routing problem proposed by Orloff [25], is coined by Cornuéjols et al. [7]. Nevertheless, at that time the authors focused only on the series-parallel graphs (SP-graphs) abstracted from the warehouses, with the goal to find a shortest tour to pick up all the orders located in different aisles in the warehouse. They extended the linear-time algorithm proposed by Ratliff et al. [29] to all SP-graphs. More results on the sTSP with applications in order-picking are summarized in [9]. For the application in road networks, Fleischmann presented a cutting plane procedure to solve instances containing as many as 292 cities [10]. Recently, Letchford et al. presented several new compact polynomial-size formulations for the sTSP instances [22], and verified that, using the best of the formulations in the CPLEX branch-and-bound solver, one can quickly solve instances of over 200 destinations. Since a vertex in the graph can be visited by the salesman multiple times if needed, the weight of an edge can be re-defined as the weight of the shortest path connecting the two ending vertices, and thus the edge weights in the sTSP always satisfy the triangle inequality. The best deterministic approximation ratio for the sTSP is 1.5 too.

For many applications of the sTSP, the travel time between two vertices could be uncertain – the travel time could be *real-time* or could be *probabilistic* information. When the travel time is real-time information, the salesman keeps receiving the updated traffic information during the traversal and has to re-plan the routing based on the new information. Different re-planning frequencies such as periodically or continuously and different objectives such as exact or heuristic or approximation algorithms have resulted in various kinds of *adaptive* algorithms [11,6,4,27,35]. When the travel time is probabilistic, the objective is accordingly to either minimize the expected total travel time [21,18,16] or to maximize the probability of all deliveries done before a given deadline [17]. The readers may refer to the surveys [13,27] for more details. A comparative study by Cheong and White [4] shows both the values and the challenges of the real-time and the probabilistic traffic information.

When the online edge blockages can be broadcasted/forecasted to the salesman when he is within a certain distance, we call this kind of edge blockages as the online *advanced* edge blockages. In typical urban road networks, short-range road blockages can indeed be predicted well, and the salesman is able to obtain them within a certain distance, thus avoids some unnecessary travel time. To the best of our knowledge, there is little research on any routing problems with online advanced traffic information; we will study in this paper the sTSP with online advanced edge blockages.

In another online variation of the TSP, a *request* is a message asking the salesman to visit a certain vertex, and it is represented as a triple (reveal time, release time, expiry time); the reveal time is the time the salesman receives the request, the release time is the beginning time the target vertex can be serviced, and the expiry time is the time after which the vertex does not have to be visited any more, but the salesman fails on this request. When the reveal times are not all zero, the requests are online [1]; when the reveal time of a request is earlier than the release time, this is an advanced request. Tsitsiklis [30] and Psaraftis et al. [28] investigated the offline version, that is the reveal times of all the requests are zero. When all the requests are revealed to the salesman a positive amount of time in advance than the respective release time, and they do not have expiry time, Jaillet and Wagner [14] presented online algorithms achieving strictly better competitive ratios than the case where the requests are not revealed in advance (that is, the reveal time of each request is the same as the release time). Wen et al. [32] studied the online TSP with advanced requests of finite expiry time. The interested readers may refer to the survey by Jaillet and Wagner [15] for more details on the TSP with online advanced requests.

In one of the earliest works on the TSP with online requests, Ausiello et al. proposed to use the *homing* TSP to refer to the standard definition of the TSP in which the salesman needs to return to the starting vertex, while the *nomadic* TSP to refer to the variant in which the salesman does not need to return to the starting vertex [1]. In this paper, we use these two terminologies in the same way.

A related routing problem with online edge blockages is the *Canadian traveler problem* (CTP), introduced by Papadimitriou and Yannakakis [26]. In CTP, the goal is to find a shortest path from a source vertex s to a destination vertex t in a given edge-weighted graph $G=(V,E)$, while edges could be blocked during the traversal from s to t. Papadimitriou and Yannakakis showed that it is PSPACE-complete to find an online algorithm with a constant competitive ratio [26]. When the number of blocked edges is known to be upper bounded by k, the variation is denoted as k-CTP. Online approximation algorithms with proven competitive ratios have been proposed for k-CTP and several other CTP variations [2,37,33,34,36].

Recently, Liao and Huang considered another variation of online TSP, called *covering Canadian traveler problem* (cCTP) [23], which is the closest to our target problem sTSP. In cCTP, a complete edge-weighted graph satisfying the triangle inequality is given, and the traveler needs to visit all vertices and then return to the origin; the blocked edges in cCTP are generated by the adversary in a different way than in sTSP – all blocked edges incident at a vertex are revealed to the traveler when he arrives at the vertex, and no more edge incident at the vertex can be blocked afterwards. Liao and Huang presented an efficient $O(\sqrt{k})$ touring strategy when the number of edge blockages is up to k [23].