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A new neighborhood structure for round robin scheduling problems

Tiago Januario, Sebastián Urrutia *

Department of Computer Science, Universidade Federal de Minas Gerais, Avenida Antônio Carlos, 6627, Belo Horizonte, 31270-010 Minas Gerais, Brazil

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ABSTRACT

In recent years, several works proposed local search heuristics for round robin sport scheduling problems. It is known that the neighborhood structures used in those works do not fully connect the solution space. The aim of this paper is to present a novel neighborhood structure for single round robin sport scheduling problems. The neighborhood structure is described in graph theory terms and its correctness is proven. We show that the new neighborhood structure increases the connectivity of the solution space when compared to previous neighborhood structures. We evaluate its performance using the Traveling Tournament Problem with Predefined Venues and the Weighted Carry-Over Effects Value Minimization Problem as case studies.

1. Introduction

In a *Single Round Robin* (SRR) tournament among an even number n of teams, each team plays against each other once over $n-1$ rounds. To model such tournament, we label the teams by $\{v_1, v_2, \ldots, v_n\}$, and represent the competition by a complete graph: the vertices represent the n teams, and each edge $e=(v_i, v_j)$ represents the match in which teams v_i and v_j play against each other.

A *schedule* of a SRR tournament is an assignment of a round to each game in the tournament such that every team plays exactly once in each round. In the proposed model a schedule is a partition of the edge set of the graph into rounds.

A *one-factor* of graph $G=(V,E)$ is a set of edges $R \subseteq E$, such that all vertices in V have degree equal to 1 in the subgraph $G'=(V,R)$ (R is also called a *perfect matching*). A *one-factorization* of $G(V,E)$ is a partition of E into one-factors, i.e., a set $R=\{r_1, r_2, \ldots, r_{n-1}\}$ of disjoint one-factors such that $\bigcup_{i=1}^{n-1} r_i = E$. Note that one-factorization and minimum proper edge coloring are interchangeable concepts for complete graphs with an even number of vertices, e.g., a one-factor f_c represents the edge-set wherein all edges are colored with color c.

As noted in [4,24], schedules of SRR tournaments have a one to one relation with *ordered one-factorizations* of complete graphs K_n in which each one-factor represents a round. Fig. 1 gives an SRR schedule represented by an ordered one-factorization of K_4. If we consider the relation between the factors and rounds $R=\{(\textit{dashed line}, \textit{round } 1), (\textit{solid line}, \textit{round } 2), (\textit{double line}, \textit{round } 3)\}$, the matches in the first round will be team 1 against team 4 and team 2 against team 3.

Two one-factorizations F and H of G, say $F=\{f_1, f_2, \ldots, f_k\}$, $H=\{h_1, h_2, \ldots, h_k\}$, are called *isomorphic* if there exists a map φ from the vertex-set of G onto itself such that $\{f_1\varphi, f_2\varphi, \ldots, f_k\varphi\}=\{h_1, h_2, \ldots, h_k\}$. Here $f_i\varphi$ is the set of all edges $(x\varphi, y\varphi)$ where (x,y) is an edge in F. The concept of *isomorphic one-factorizations* can also be extended to *isomorphic schedules* and *isomorphic colorings*, therefore two schedules (resp. colorings) are said to be isomorphic if, and only if, their associated one-factorizations are isomorphic.

A one-factorization is said to be perfect, and called *perfect one-factorization*, when the graph induced by the edges in $f_i \cup f_j$ ($G(V, f_i \cup f_j)$) is a *Hamiltonian cycle* for each pair of distinct one-factors f_i and f_j.

Let s be a schedule of an SRR tournament with n teams and let the solution space $\mathcal{S}$ be the set of all possible schedules for those n teams. A *neighborhood structure* is a mapping that assigns to each schedule $s \in \mathcal{S}$, a set of schedules $\mathcal{N}(s)$ that are neighbors of s. Local search procedures use the concept of neighborhoods to move from one schedule s to a neighbor schedule $s' \in \mathcal{N}(s)$. In this work, all neighborhood structures will be modeled in terms of operators over ordered one-factorizations (proper edge colorings with minimum number of colors) of complete graphs.

Different neighborhood structures have been used in local search procedures for round robin sport scheduling problems in the literature, as can be found in Anagnostopoulos et al. [2], Costa et al. [3], Di Gaspero and Schaerf [5], and Ribeiro and Urrutia [20]. The nomenclature of the existing neighborhood structures is not very consistent, but several articles called them *Team Swap*, *Partial Team Swap*, *Round Swap* and *Partial Round Swap*.

* Corresponding author. Tel.: +55 31 3409 5860.
E-mail addresses: januario@dcc.ufmg.br (T. Januario), surrutia@dcc.ufmg.br (S. Urrutia).

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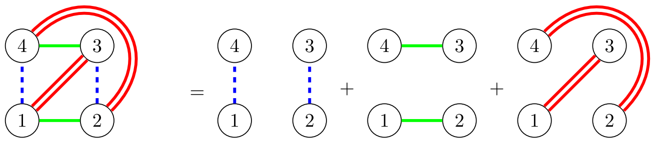

Fig. 1. An example of a tournament represented by an ordered one-factorization of K_4.

It was established in Costa et al. [3] that the solution space of SRR tournaments with specific number of participating teams is not connected by those neighborhood structures, which explains the hardness of finding high quality solutions to some problem instances. In Januario and Urrutia [12] the authors extended the research started in Costa et al. [3] by investigating the sizes of tournaments for which the neighborhoods are not connected.

In this paper, we propose a novel neighborhood structure for SRR sport scheduling problems to be used by local search procedures: *Teams and Rounds Swap* (TARS). Using two problems from the literature as case studies, we show that this neighborhood structure diminishes the issue of non-connectivity in the existing ones for SRR scheduling problems. Moreover, the use of TARS can lead to good results for both case study problems regardless of the initial solution or the problem instance.

We recognize the importance of other tournament structures as double round robin and other problem classes that take into account, for example, the home-away assignment for each game. However, in this paper we consider just SRR tournaments without home-away assignments. We note that our results on the plain SRR tournament format may be adapted to other types of tournament formats. Extensive literature reviews about sport scheduling problems, and their applications may be found in [13,19].

The rest of this paper is organized as follows. Section 2 introduces the existing neighborhood structures for round robin sport scheduling problems. A detailed description of the proposed TARS neighborhood structure followed by the proof of its correctness is given in Section 3. Next, a discussion about how TARS increases the solution space connectivity is given in Section 4. An experimental analysis of the TARS neighborhood structure using the Traveling Tournament Problem with Predefined Venues and the Weighted Carry-Over Effects Value Minimization Problem as case studies is performed in Section 5. In the last section, we state some concluding remarks.

2. Existing neighborhood structures

In this section, all neighborhood structures are described as functions that operate on edge colored graphs. Each new coloring obtained by each function is a neighbor of the current schedule in the neighborhood structure under consideration. Let $c(v,w)$ be the color assigned to edge (v,w) and let $adj(v,d)$ be the vertex w such that $c(v,w)=d$.

Each neighbor in the Team Swap (TS) neighborhood is obtained by swapping two distinct vertices v and w in the graph. After a TS move, v has the assignment of colors to its incident edges that previously belonged to w and vice-versa. Fig. 2 illustrates the application of TS, using vertices 3 and 4 as parameters, based on a schedule with six teams.

In order to obtain a neighbor in a Round Swap (RS) neighborhood we take two distinct used colors, c_j and c_k, and swap them for all edges colored with any of those colors, i.e., we swap the two factors, r_j and r_k, associated to the colors c_j and c_k. Fig. 3 illustrates the application of RS, using the colors represented by the double solid lines and double dashed lines as move parameters, based on a schedule with six teams.

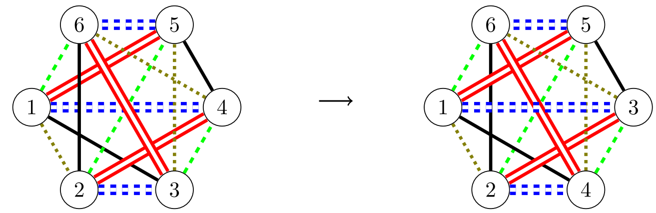

Fig. 2. *On the left*: an ordered one-factorization of K_6. *On the right*: a graph obtained through a Team Swap move using vertices 3 and 4 as move parameters.

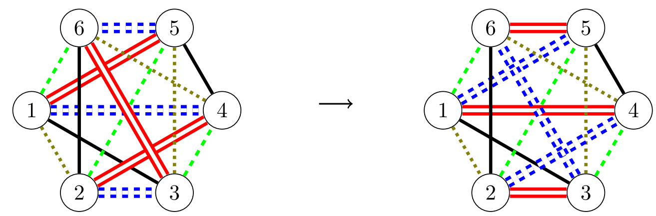

Fig. 3. *On the left*: an ordered one-factorization of K_6. *On the right*: a graph obtained trough a Round Swap move using the colors represented by the double solid lines and double dashed lines as move parameters.

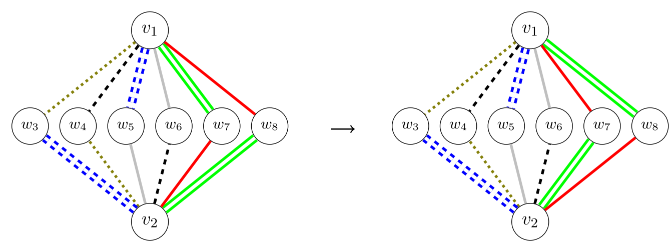

Fig. 4. *On the left*: a subgraph obtained from an ordered one-factorization of K_8. *On the right*: the same subgraph after a Partial Team Swap move, such that $W=\{w_7,w_8\}$ and $\Omega=\{c(v_1,w_7),c(v_1,w_8)\}=\{c(v_2,w_7),c(v_2,w_8)\}$.

The Partial Team Swap (PTS) neighborhood structure is a generalization of the TS neighborhood structure. Let C be the set of all colors in $G(V,E)$. For any color c and for any two distinct vertices v_1 and v_2, with $c\neq c(v_1,v_2)$, let Ω be a minimum cardinality subset of colors including c in which the set of vertices connected to v_1 by an edge colored with a color in Ω are the same as those connected to v_2 by an edge colored with a color in Ω. That is, $\Omega=\{c_1,\ldots,c_k\}\subseteq C$ is minimum and such that $c\in\Omega$ and $W=\{w\in V:\exists c_j\in\Omega$ such that $c(w,v_1)=c_j\}=\{w\in V:\exists c_j\in\Omega$ such that $c(w,v_2)=c_j\}$. A neighbor is obtained by swapping the colors of (v_1,w) and (v_2,w) for all $w\in W$, see Fig. 4. If Ω is equal to $C\backslash c(v_1,v_2)$ then the move in PTS is equivalent to a move in TS.

Swapping the colors of (v_1,w) and (v_2,w) is a constant time operation for each $w\in W$. We can trivially bound the worst-case of the number of vertices involved in the move to $O(n)$. Therefore, for a given set of parameters, a move in a TS or a PTS neighborhood can be performed in $O(n)$ in the worst-case scenario.

Finally, for a move in the Partial Round Swap (PRS) neighborhood structure, we select any two distinct colors, for instance c_1 and c_2, and consider any cycle in the subgraph induced by the edges colored with those colors. Then, we swap the colors in that cycle to get a neighbor schedule. If the involved edges are incident to all the vertices of the graph, i.e., if they form a Hamiltonian cycle, then this move is equivalent to a move in the RS neighborhood structure. As a consequence, PRS and RS are equivalent for perfect one-factorizations. Fig. 5 gives an illustration of a move in the PRS neighborhood.