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Finding compromise solutions in project portfolio selection with multiple experts by inverse optimization

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ABSTRACT

This paper deals with project portfolio selection evaluated by multiple experts. The problem consists of selecting a subset of projects that satisfies a set of constraints and represents a compromise among the group of experts. It can be modeled as a multi-objective combinatorial optimization problem and solved by two procedures based on inverse optimization. It requires to find a minimal adjustment of the expert's evaluations such that a portfolio becomes ideal in the objective space. Several distance functions are considered to define a measure of the adjustment. The two procedures are applied to randomly generated instances of the knapsack problem and computational results are reported. Finally, two illustrative examples are analyzed and several theoretical properties are proved.

1. Introduction

Project portfolio selection is a common problem that frequently includes the evaluation of each project by multiple experts [1,2]. It requires to select a subset of projects that satisfies a set of constraints and represents a compromise among the group of experts. Traditional constraints are budget satisfaction and project dependencies.

In this paper, it is assumed that the expert's evaluations are of a quantitative nature as illustrated in the following example. Consider a company that engages a call for proposals for R&D projects that would be operated in the forthcoming years. Each project is then evaluated by estimating its net present value (NPV). However, this quantity is non-unique, because it requires fixing the rate of return and the period to observe. For example, it might be calculated over a week or a month. As a result, a group of experts is hired to asses the proposals. It is decided that each expert can choose how the net present value is evaluated. Based on these evaluations, the portfolio must maximize the net present value of each expert and satisfy a budget constraint. This decision making task is modeled as a multi-objective combinatorial optimization problem, where each objective function is an expert's evaluation.

A usual approach to tackle this problem consists of building a utility function to quantify the performance of each portfolio. A simple way to build such a utility function would be to aggregate the experts evaluations by the geometric or the arithmetic mean [3–5]. In these cases, the evaluations of each expert are aggregated in such a way that the group may be seen as a new "individual". This leads to a classical project portfolio problem where only one decision maker is involved.

Another way to deal with this problem is to model it as a multi-objective combinatorial optimization problem where each objective function is an expert's point of view over the portfolios. This judgment is the sum of the expert's evaluation of the projects selected in the portfolio. A portfolio is said "ideal" if it maximizes the judgments of all experts. Let us observe that if there exists an ideal portfolio, then there is a consensus among the experts on this portfolio. Based on this observations, several concepts of compromise portfolios can be defined. The first one is to find a portfolio (or a set of portfolios) that is as close as possible to the ideal portfolio [6]. This leads to a set of compromises, which depend on the choice of a distance function. Let us consider the efficient set and the ideal solution in Fig. 1. In this example, the compromise solution with respect to the Euclidean distance is v_3, because it is the closest solution to $\bar{f}$.

A second concept of compromise is presented in this paper. It is based on the following observation: evaluations are not necessarily precise and a slight modification of their value could be accepted. Hence, the compromise solution may be determined by finding a minimal adjustment of the experts' evaluations so that an ideal portfolio exists. This leads to a new multi-objective optimization model that is as close as possible to the original one, where there exists an ideal solution. For example, this could lead to transform the problem of Fig. 1 into the problem of Fig. 2, where v_2 is the compromise solution. Our approach also allows, in a

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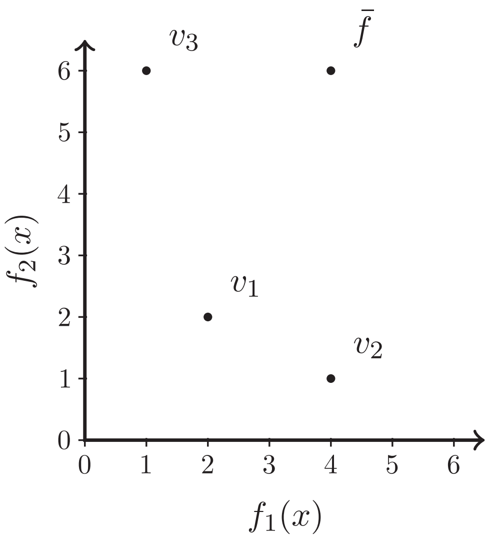

Fig. 1. A set of efficient solutions $\{v_1, v_2, v_3\}$ and the ideal outcome vector $\bar{f}$.

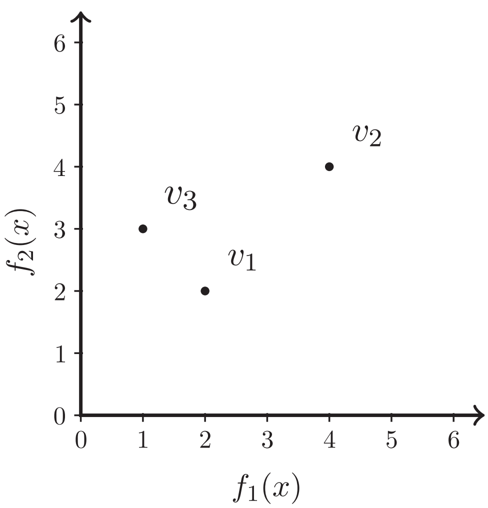

Fig. 2. A set of feasible solutions $\{v_1, v_3\}$ and the ideal solution v_2.

certain sense, to measure how the experts are conflicting with each other and could indicate potential conflicts among experts on the evaluations of the projects.

This concept of compromise is closely related to inverse multi-objective optimization [7]. It is defined as the minimal adjustment of the problem parameters inducing a change (addition or removal of solutions) in the efficient set. Indeed, it is easy to see that a compromise can be obtained through the minimal adjustment of the parameters, in such a way that the cardinality of the image of the efficient set in the objective space is equal to one. However, this precise question has not been covered yet in terms of inverse multi-objective optimization. This constitutes the purpose of the paper.

Two approaches are considered for finding a compromise solution. The first one consists of solving an inverse problem, by a cutting plane approach, for each efficient solution. The second one consists of solving iteratively a particular linear integer program.

The proposed methods are studied from a theoretical and a practical point of view. Several properties, such as influence of non-discriminating experts, monotonicity, and dominance are proved to be satisfied. Then, the compromise solutions of an illustrative example are analyzed and compared to the ones obtained by the Zeleny's procedure.

This paper is organized as follows. In Section 2, concepts, definitions, and notation are introduced. In Section 3, the inverse problem is formally defined and theoretical results are provided. Two algorithms are proposed for finding a compromise solution by inverse optimization in Section 3.1. The design of the experiments and computational results are presented in Section 4. In Section 5, an illustrative example is analyzed. We conclude with remarks and directions for future research.

2. Concepts, definitions, and notation

Let $\mathbb{R}^n = \{(x_1, x_2, \ldots, x_j, \ldots, x_n) : x_j \in \mathbb{R}$ denote the set of real-valued vectors of length $n \geq 1$, for $j \in J = \{1, 2, \ldots, n\}$, and $\mathbb{N} = \{0, 1, 2, \ldots\}$ the set of non-negative integers. A vector $x \in \mathbb{R}^n$ is a matrix composed of 1 column and n rows, and the transpose of x, denoted by $x^\top$, is a matrix composed of n columns and 1 row.

Multi-objective optimization consists of maximizing "simultaneously" several objective functions over a set of feasible solutions. The *feasible set* is denoted by $X \subseteq \mathbb{R}^n$. The outcome of each feasible solution $x \in X$ is denoted by a vector $F(x) = (f_1(x), f_2(x), \ldots, f_i(x), \ldots, f_q(x))$ composed of the outcomes of the q objective functions $f_i : X \to \mathbb{R}$, with $i \in I$, where $I = \{1, 2, \ldots, q\}$ is the set of objective subscripts.

A particular class of Multi-Objective Combinatorial Optimization problems (MOCO) is considered. Each instance is defined by a pair (X, C) where $X \subseteq \{x : x \in \{0, 1\}^n\}$ is the feasible set and $C \in \mathbb{N}^{q \times n}$ is the so-called *profit matrix*. Each objective function $f_i : X \to \mathbb{N}$, with $i \in I$, is defined by a row of the profit matrix with $f_i(x) = \sum_{j \in J} C_{ij} x_j$.

Let $x, y \in \mathbb{R}^n$ be two vectors. The following notation will be used hereafter: $x < y$ iff $\forall j \in J : x_j < y_j$; $x \leqq y$ iff $\forall j \in J : x_j \leq y_j$; $x \neq y$ iff $\exists j \in J : x_j \neq y_j$; $x \leq y$ iff $x \leqq y$ and $x \neq y$. The binary relations $\geqq$, $\geq$, and $>$ are defined in a similar way.

In multi-objective optimization, two spaces should be distinguished. The *decision space*, *i.e.*, the space in which the feasible solutions are defined, and the *objective space*, *i.e.*, the space in which the *outcome vectors* are defined. The image of the feasible set in the objective space is denoted by $Y = \{y \in \mathbb{R}^q : y = Cx, x \in X\}$.

An ideal outcome vector $y^* \in \mathbb{R}^q$, to an instance (X, C), is defined by $y_i = \max\{\sum_{j=1}^n C_{ij} x_j : x \in X\}$, for all $i \in I$. A feasible solution $x \in X$ is said to be ideal if and only if Cx is an ideal outcome vector. Such a solution does not always exists. Consequently, it is widely accepted to build the dominance relation on the set Y of the outcome vectors. Let $y, y' \in Y$ be two outcome vectors such that $y \neq y'$. It is said that y dominates y' if and only if $y \geq y'$. *Dominance* is a binary relation that is irreflexive, asymmetric, and transitive. This relation induces a partition of Y into two subsets: the set of dominated outcome vectors and the set of *non-dominated* outcome vectors. The set of non-dominated outcomes corresponding to an instance (X, C) is denoted by $ND(X, C)$. Similarly, in the decision space the concepts of efficient and non-efficient solutions can be defined. A solution $x^* \in X$ is *efficient* if and only if there is no $x \in X$ such that $Cx \geq Cx^*$. The set of efficient solutions corresponding to an instance (X, C) is denoted by $E(X, C)$.

Let $I = \{1, 2, \ldots, i, \ldots, q\}$ denote a set of experts, and $J = \{1, 2, \ldots, j, \ldots, n\}$ a set of items. The evaluation of an expert $i \in I$ over each item $j \in J$ is denoted by $C_{ij} \in \mathbb{N}$. The portfolio selection problem (PSP) consists of selecting a subset $S \subseteq J$, such that the sum of the evaluations of the elements belonging to S is "maximized" and simultaneously satisfies a set of constraints. The set of constraints is defined by the system $Ax \leqq b$, where $x \in \{0, 1\}^n$ is the incidence vector of S, $A \in \mathbb{R}^{m \times n}$, and $b \in \mathbb{R}^m$. The problem can be stated as follows:

$$\text{"max"} \quad F(x) = \{f_1(x), f_2(x), \ldots, f_i(x), \ldots, f_q(x)\}$$