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The robust (minmax regret) single machine scheduling with interval processing times and total weighted completion time objective

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ABSTRACT

Single machine scheduling is a classical optimization problem that depicts multiple real life systems in which a single resource (the machine) represents the whole system or the bottleneck operation of the system. In this paper we consider the problem under a weighted completion time performance metric in which the processing time of the tasks to perform (the jobs) are uncertain, but can only take values from closed intervals. The objective is then to find a solution that minimizes the maximum absolute regret for any possible realization of the processing times. We present an exact branch-and-bound method to solve the problem, and conduct a computational experiment to ascertain the possibilities and limitations of the proposed method. The results show that the algorithm is able to optimally solve instances of moderate size (25–40 jobs depending on the characteristics of the instance).

1. Introduction

Single machine scheduling is a classical problem thoroughly studied in the literature (see [9,12,16,17,26,33,34] among others). The objective of the problem is to ascertain the optimal sequence in which the tasks (jobs) that define the problem need to be performed in a machine, in order to optimize some performance measure. In addition to its theoretical interest, and its use on real life situations in which only one resource (machine) exists, the problem also models situations in which the production system is taken as a whole (hence, it is considered to be a unity) or there is a bottleneck operation that dictates the performance of the production system.

While the most studied formulations correspond to deterministic problems [9,33,34,17] that is, formulations in which all of the parameters are known and fixed, formulations with different characterizations of the uncertainty have also been studied [30,35,26,14,2,5].

Among the different alternatives to model uncertainty, we highlight the use of probability distribution functions to describe processing times [35,14,5], the representation of possible processing times through scenarios [10,18], and the use of intervals to define the processing time of the jobs [11,30,2].

In this paper we study an interval data minmax regret (IDMR) (see [11,1,30]) formulation of the single machine weighted sum of completion times problem (WCTP) [29,26]. In this formulation, the uncertainty on the processing time is represented using intervals, and the aim is to find a sequence that minimizes the maximum regret for all of the possible scenarios (that is, the sequence that minimizes the maximum absolute deviation between its solution and the optimal solution for each realization of processing times). Hence, the objective reflects a robust criterion which can be associated to a risk-averse decision maker that tries to hedge against the worst-case performance. This problem is referred as the Robust Weighted Completion Time Problem (RWCTP) throughout the paper.

1.1. Contributions of this work

While the robust minmax regret sum of completion times problem has been studied in the literature (see [11,19,37,20,15]); to the best of our knowledge its weighted counterpart has not been previously considered. We conjecture that the cause for this lack of research directed toward its resolution is the inapplicability of the classical results used to evaluate a solution in multiple IDMR problems (see, e.g. [1]), which leads to an absence of a viable method to evaluate the maximum regret of a given sequence.

In this paper we deal with the previous issue and we propose two different methods to evaluate the maximum regret of any given solution: one based on enumeration, and a second based on a dynamic programming (DP) formulation [6]. Both methods build

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upon the identification of a subset of scenarios containing the worst case scenario. Note that this approach differs from other studied IDMR problems in which the worst case scenario and/or the worst case alternative are obtained by solving some classical optimization problem.

Then, a branch-and-bound procedure using a novel lower bound method and a previously known dominance rule [30] is used to enumerate all of the solutions. The method is initialized using a heuristic which is shown to provide a 2-approximation, and if the lower bound or the dominance rule are not able to discard a solution during enumeration, the maximum regret of the said solution is obtained using one of the methods described above.

A computational experiment using instances from previous works on other weighted completion time problems with interval data [30,2] shows that the proposed branch-and-bound is capable of optimally solving instances with up to 25 jobs if the variability of processing times is high, and instances with up to 40 jobs if the variability is low. Furthermore, the applicability and the approximation ratio of a simple heuristic for IDMR problems, the midpoint scenario heuristic (see for example [19,15]) is also considered.

Note that the size of these instances is smaller than the size of the instances solvable for the unweighted case, in which larger instances can be solved to optimality (see [23,8]). Also note that the RWCTP includes an additional level of complexity, as the objective function depends not only on the absolute but also on the relative position of each pair of jobs. Nevertheless, the proposed branch-and-bound method is able to solve medium sized instances to optimality within limited running times. The results for the midpoint scenario heuristic in optimally solved instances show that it provides very reduced optimality gaps, and thus it appears as a good alternative if the solution of larger instances is required.

1.2. Paper outline

The rest of the paper is structured as follows. In Section 2 we describe the problem; we study the previous work on related problems and we put forward some notation used in the following sections. Section 3 is devoted to the contributions of this work and the proposed branch-and-bound algorithm. Section 4 sets forth a computational experiment to evaluate the efficacy of the algorithm proposed in Section 3. Finally, Section 5 puts forward the conclusions of this work. Two appendices are included: Appendix A is devoted to the approximation guarantees of the midpoint scenario heuristic for the RWCTP; and Appendix B, which can be found in the electronic Supplementary Material, provides additional results from the computational experiment reported in Section 4.

2. Problem settings

2.1. Description of the deterministic problem

The one machine scheduling problem with weighted sum of completion times (WCTP) is a classical problem, see for example Pinedo [26]. According to the notation proposed in Graham et al. [12], the problem corresponds to $1 \| \sum w_j C_j$, and it can be formally described as follows: n independent jobs $J = \{J_1, J_2, \ldots, J_n\}$ have to be processed over a single machine that can handle at most one job at a time. Each job J_i, $1 \le i \le n$, is available for processing at time zero, requires a processing time $p_i > 0$ and has a positive weight $w_i > 0$. Let C_i be the completion time of job i, then the objective is to obtain a processing order, a sequence, for the jobs such that the weighted completion time of the jobs, that is $\sum_{i \in J} w_i C_i$, is minimized.

This problem is easily solvable as the optimal sequence is obtained by sorting jobs according to their weighted shortest processing times (WSPT), see Smith [29].

2.2. Description of the robust (minmax regret) problem

The Robust (minmax regret) weighted sum of completion time problem (RWCTP) departs from the WCTP formulation by considering that the processing time p_i of each of the jobs is unknown before scheduling but bounded between a given lower p_i^- and an upper p_i^+ value. As the processing times are not known in advance, the objective is to minimize the maximum regret of the chosen sequence.

We proceed to formalize the objective and the concept of regret, as well as other terminology which will be used.

Let S be the set of all of the permutations (sequences). For any given sequence σ ($\sigma \in S$), we denote as $\sigma(k)$ the job occupying position k in sequence σ and as $\pi_\sigma(i)$ the position occupied by job J_i in the sequence. Each possible realization of processing times is known as a scenario ($p \in P$). Throughout the paper we refer to a scenario in which either $p_i = p_i^-$ or $p_i = p_i^+$ holds for every job, as an extreme scenario. Let us also denote the completion time of job i under scenario p as $C_i(\sigma, p)$. Then, expression (1) provides the total weighted completion time of sequence σ

$$\sum_{i=1}^{n} w_i \cdot C_i(\sigma, p) \tag{1}$$

Let σ_p^* be the sequence that minimizes (1) for a given scenario p. Then, the regret of a sequence σ under scenario p corresponds to $Z_p(\sigma)$

$$Z_p(\sigma) = \sum_{i=1}^{n} w_i \cdot C_i(\sigma, p) - \sum_{i=1}^{n} w_i \cdot C_i(\sigma_p^*, p) \tag{2}$$

The scenario $p \in P$ that maximizes (2) for any given sequence σ is known as the worst-case scenario of σ; its regret is denoted by $Z(\sigma)$; and its optimal sequence σ_p^* as the worst-case alternative of σ

$$Z(\sigma) = \max_{p \in P} \left(\sum_{i=1}^{n} w_i C_i(\sigma, p) - \sum_{i=1}^{n} w_i C_i(\sigma_p^*, p) \right) \tag{3}$$

The objective of the RWCTP is to obtain the sequence that minimizes the maximum regret for all possible scenarios, that is, the sequence σ that minimizes Z

$$Z = \min_{\sigma \in S} Z(\sigma). \tag{4}$$

Note that optimally solving (4) does not only imply finding a sequence σ that minimizes (4), but also finding its worst case scenario, a realization of processing times p, and its worst case alternative σ_p^*, see (2).

2.3. Literature review

Several IDRM scheduling problems have been studied in the literature. The robust total completion time problem, the unweighted counterpart of the RWCTP, was considered in Daniels and Kouvelis [11], Kouvelis and Yu [19], Lebedev and Averbakh [20], Yang and Yu [37], and Kasperski and Zielinski [15].

Daniels and Kouvelis [11] and Kouvelis and Yu [19] show that the absolute regret problem is NP-hard, and propose a branch-and-bound for its resolution. The method uses a surrogate relaxation bound and it is able to solve instances with up to 20 jobs within short running times (less than 10 min). These results