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Valuation of commodity derivatives with an unobservable convenience yield

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ABSTRACT

This paper extends the existing literature on commodity derivatives to account for an unobservable stochastic convenience yield. Investors operate in an economy with incomplete information. In contrast to other incomplete information models, analytical formulas for forward and futures prices, as well as for European options on forward and futures contracts are obtained. These formulas reveal the important role played by the initial values of the estimator of the convenience yield and of the estimation error respectively when valuing commodity derivatives. We estimate Schwartz's [11] model and the incomplete information model based on the discrete-time Kalman filtering method. For futures prices, the latter seems to perform better than the former. Moreover, Schwartz's model provides higher option prices than the incomplete information model. The most important differences are obtained for higher futures prices and for longer options maturities.

1. Introduction

In recent years, storable commodity markets (energy, agricultural, metals, etc.)[1] booms and busts have revived the interest for commodity derivatives, which have been characterized by a dramatic growth in trading volume and by the proliferation of contracts written on a wide range of underlying commodities. Especially, according to the Bank of International Settlements (BIS), the notional value of over-the-counter (OTC) commodity derivatives stood at $2.6 trillion in 2012. In the US and in the European Union regulation in force has a significant impact on the commodities trading industry and on OTC contracts, in particular. This may have important implications on pricing, hedging and managing risk in commodity derivatives markets.

Not surprisingly, much literature has been devoted to the pricing and hedging of commodity derivatives. Reduced-form models turn out to be the appropriate models for such considerations. They try to identify the relevant state variables or factors associated with the dynamics of the futures prices. The stochastic processes of these variables are specified exogenously by taking into account some of the specific features describing the behavior of commodities. Storable commodities, in contrast to other conventional securities, are real assets that can be produced and consumed. Thus holding physical inventories may provide some services in order to avoid shortages and therefore to maintain the production process or to respond to an unexpected demand, for example.[2] These services are qualified to as the convenience yield, when expressed as a rate (see, for example, the definition in [1]). The theory of storage [2–4] has studied the relationship between commodity prices and storage decisions, and has established a no-arbitrage relation between futures prices, spot commodity prices, the interest rate and the net convenience yield. Moreover, a growing number of empirical studies on commodity return predictability pointed out the important role of the convenience yield especially (see, for instance [5–10]). The spot price and the convenience yield are therefore the two commonly used state variables in pricing models.

It is widely recognized in the literature (see, for instance [11]) that the convenience yield is not directly observable. Thus, when pricing commodity derivatives, a difficulty arises from this non-observability. Gibson and Schwartz [12] used the theory of storage

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[1] Typical examples of non-storable commodities include livestock, electricity and perishable commodities.

[2] Gülpinar et al. [14], for instance, study the impact of supply disruptions in the oil market on portfolio management involving commodity derivatives.

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formula to calculate the implied convenience yield. The latter can be inferred by using futures and spot prices (or the futures contract price closest to maturity as a proxy for the spot price if it is not available). However, prices may be subject to temporary mispricing or to liquidity concerns, for example. Carmona and Ludkovski [13] also showed that the implied convenience yield is unstable over time and incompatible with the forward curve. Using prices from futures contracts with different maturities will provide very different convenience yields. Dockner et al. [15] suggest a new analytical approximation of the convenience yield by computing the difference between the present values of two floating-strike Asian options written on the spot and the futures prices, respectively. They have found, however, that different approximation methods give rise to different convenience yield estimates. The differences can be attributed to the cost of carry and the moneyness of the options.

The main objective of this paper is to build a reduced-form valuation model of commodity forward and futures contracts as well as options on forward and futures contracts by explicitly taking into account the unobservable character of the convenience yield. Although many models dealing with a partially observable economy or with incomplete information have studied the unobservability of a stochastic expected return (see, for instance [16–18] for an application to interest rates and to asset allocation), to the best of our knowledge, the case of an application to commodities with a partially observable convenience yield has not been examined yet. A notable exception is Carmona and Ludkovski [19] who developed a utility-based pricing model resulting in a partially observable stochastic control problem.

The reduced-form models fall into two categories. In the first one, in addition to the spot price and the convenience yield (see [12,20,21], and Liu and Tang [22]), other state variables and some stylized facts have been suggested in the literature. The interest rate [11,23,24], the volatility of the spot price [25–27], and Benth [28], jumps in the spot price and the volatility [23,25], and Schmitz et al. [29] or seasonal patterns in commodity prices [26,30,31], and Back et al. [32]. Schwartz and Smith [33] have suggested a two-factor model allowing for a short-term mean reversion in spot prices toward a random long-term equilibrium level. To take into account some observed features of the convenience yield, Paschke and Prokopczuk [34] have generalized the Schwartz and Smith model by assuming that the short-term component follows a continuous autoregressive moving-average process. Garcia et al. [36] have developed a four-factor model to take into account mean reversion and stochastic seasonality in the convenience yield. Cortazar and Naranjo [37] and Paschke and Prokopczuk [38] have developed multi-factor models to explain the random behavior of oil prices. The second category of the reduced-form models is inspired by the Heath et al. [39] model, which is consistent with the initial term structure of interest rates. It was pioneered by Miltersen and Schwartz [40] who built a model to price options on commodity futures, with a stochastic convenience yield, which is consistent with the initial term structures of both interest rates and commodity futures prices. Miltersen [41] have extended the Gibson and Schwartz [12] model in order to fit the initial term structure of forward and futures prices, as well as that of forward and futures volatilities.[3] Trolle and Schwartz [42] have suggested a model to account for (unspanned) stochastic volatility of both the spot price and the instantaneous forward cost of carry. Manoliu and Tompaidis [43] have proposed a multi-factor model accounting for seasonal patterns in the futures curve.

The models mentioned above are constructed in a fully observable framework. They assume that the variables which describe the state of the economy are observable. However, some of the state variables, which are relevant to financial claims valuation, are generally not observable, the instantaneous convenience yield, in particular. Investors operate in an economic environment with incomplete information. The clarifying paper of Feldman [46] has judicially discussed several issues related to the incomplete information equilibrium. Dothan and Feldman [47], Detemple [48], Gennotte [49] and Xia [18], among others, have investigated, in a dynamic framework, the optimal asset allocation and asset valuation in a partially observable economy. In this economy, the agent estimates one or more unobserved state variable(s) given information conveyed by past observations. The estimation or filtering error represents the agent's assessment of the precision of the estimate and reflects his (her) quality of information. The investor's optimization problem is then based on the conditional mean(s), which also determine(s) contingent claims prices.[4]

In this article, we propose a three-factor model in an incomplete information setting in the spirit of the Schwartz [11] model 3, which is the reference model in the literature involving the convenience yield. In order to do so, the economic framework retains the spot commodity price, the instantaneous interest rate and the convenience yield as the relevant stochastic state variables associated with the dynamics of the futures price. Contrary to this model and to the other reduced-form models, the convenience yield is not observable, but investors can draw inferences about it by observing the spot commodity price and the short rate by using the continuous-time Kalman–Bucy filter method.[5]

From a theoretical point of view, our contribution to the literature is threefold. First, we extend the relevant literature by pricing derivatives under incomplete information.[6] This framework is well-adapted to the case of storable commodities since the convenience yield is unobservable. Second, in sharp contrast to models in a partially observable economy, we provide simple closed-form solutions for forward and futures prices, as well as for European options on forward and futures contracts. Thus our model inherits this appealing feature of fully observable reduced-form models with a Gaussian structure facilitating its use for practical considerations. Also, two-factor and one-factor models can be easily derived. Third, these analytical solutions allow one to examine the impact of the model parameters on derivatives' prices. Especially, they reveal that futures and option prices under incomplete information depend on the initial values of the estimate of the convenience yield and of the estimation error respectively. Thus, we can study the influence of the incomplete information on commodity derivatives prices.

We estimate the parameters of the processes of the spot commodity price, the short rate and the convenience yield by using weekly data on U.S. Treasury bills and on West Texas Intermediate (WTI) light sweet crude oil futures contracts traded on Intercontinental Exchange (ICE) for the period 2001–2010. Since the convenience yield is not supposed to be directly observable, the estimation is based on the discrete-time Kalman filter method, which is the appropriate method when state variable are not observable and are Markovian (see, for instance [11,33,37,38,43]).

[3] This a direct analog of the Hull and White [44] model which extended the Vasicek [45] model to fit the initial yield curve and the initial term structure of the spot rate volatility.

[4] Hernandez et al. [50] have suggested an alternative procedure, a moment method algorithm, to estimate the parameters of an unobservable stochastic mean and have applied it to commodities.

[5] The Casassus and Collin-Dufresne [24] model allows the convenience yield to depend on the spot price and the interest rate, but in a fully observable economy.

[6] See also Mellios [35] who has developed a model to price European-style interest rate options in a partially observable economy.