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The stochastic lot sizing problem with piecewise linear concave ordering costs

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ABSTRACT

We address the stochastic lot sizing problem with piecewise linear concave ordering costs. The problem is very common in practice since it relates to a variety of settings involving quantity discounts, economies of scales, and use of multiple suppliers. We herein focus on implementing the (R, S) policy for the problem under consideration. This policy is appealing from a practical point of view because it completely eliminates the setup-oriented nervousness – a pervasive issue in inventory control. In this paper, we first introduce a generalized version of the (R, S) policy that accounts for piecewise linear concave ordering costs and develop a mixed integer programming formulation thereof. Then, we conduct an extensive numerical study and compare the generalized (R, S) policy against the cost-optimal generalized (s,S) policy. The results of the numerical study reveal that the (R, S) policy performs very well – yielding an average optimality gap around 1%.

1. Introduction

The need for coordination and cooperation between supply chain players has dramatically increased due to rapid progress in globalization and competition. In this context, a common complaint raised by supply chain managers is that downstream players continually revise the timing and the size of their order requests [7]. It is commonly agreed that the revisions in replenishment schedules are particularly critical [4]. This issue is often referred to as the setup-oriented nervousness and it is prevalent in a variety of industrial settings involving managing joint replenishments [13], shipment consolidation in logistics [8], and buying raw materials in markets where price is fluctuating [6]. A practical approach towards offsetting setup-oriented nervousness is to employ an (R, S) policy [1]. Here, a replenishment schedule is fixed once and for all at the beginning of the planning horizon, but the size of replenishments are dynamically determined at the time of placing orders upon observing realized demands. This strategy has recently been subject to a detailed scrutiny due to its practical relevance, and applied to a variety of inventory control problems [1,14,15,11] (see e.g.). All of these studies have analyzed the (R, S) policy under the assumption that ordering cost is comprised of a fixed and a linear component. In this study, we aim to extend this literature by presenting a mathematical programming model of the (R, S) policy for piecewise linear concave ordering costs. This cost structure is a class of decreasing average costs that captures the combination of linear and fixed ordering costs as a special case. It reflects many practical examples such as quantity discounts, effect of scale economies, and use of multiple suppliers (see e.g [9,2,17]).

The majority of research efforts on inventory problems with piecewise concave ordering costs has been concentrated on characterizing the structure of the optimal control policy. Porteus [9,10] showed that a generalized (s,S) policy is optimal for inventory systems for a class of demand distributions. Fox et al. [2] considered a specific case of piecewise linear concave ordering costs, and proved that a generalized (s,S) policy is optimal for a larger class of demand distributions. Zhang et al. [18] addressed the same problem under limited order capacities. Yu and Benjafaar [17] extended earlier results by establishing the optimality of generalized (s,S) policies for general demand distributions. These research contributions have ultimately showed that generalized (s, S) policies are optimal under a large variety of settings. However, finding the optimal parameters of the generalized (s,S) policy is still a computational challenge. Also, an important drawback of the

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generalized (s,S) policy is that it does not provide the exact timing of replenishments in advance. As such, inventory systems controlled by the generalized (s,S) policy are exposed to a great deal of setup-oriented system nervousness which results in complications on the coordination between supply chain players (see e.g. [3–5,7,16].

In this paper, we adopt the (R,S) policy for the stochastic lot sizing problem with piecewise linear concave ordering costs, and show that it is a viable alternative to the cost optimal (s,S) policy. The contribution of the current study is two-fold. First, we introduce a generalized version of the (R,S) policy for the inventory problem with piecewise linear concave ordering costs, and present a mixed-integer programming (MIP) formulation thereof. Secondly, we conduct an experimental study that compares (s,S) and (R,S) policies, and thus reflect upon the trade-off between the setup-oriented system nervousness and cost optimality in case of piecewise concave ordering costs.

The remainder of this paper is organized as follows. Section 2 provides the problem definition. Sections 3 and 4 introduce solution methods for (s,S) and (R,S) policies, respectively. Section 5 provides an illustrative numerical example for two alternative policies. Section 6 is devoted to the design and findings of the computational experiments. Finally, Section 7 draws conclusions.

2. Problem definition and preliminaries

We consider a single product periodic-review finite-horizon inventory control problem. The planning horizon is comprised of T periods. The demand ξ_t in period t is an independent and normally distributed random variable with known parameters. The demand distribution may vary from period to period (i.e., demand is non-stationary). A holding cost h is incurred for each unit carried in inventory from one period to the next, and a shortage cost p is incurred for each unit of demand backordered. The ordering cost is a piecewise linear function of the order size. For the sake of simplicity, the delivery lead time is assumed to be negligible.

We make use of a multi-supplier inventory control system in order to ease the exposition of piecewise linear concave ordering costs. Assume that we have N suppliers with different cost structures. Placing an order from supplier n incurs a fixed ordering cost K^n, and a variable unit cost v^n. We assume that $K^1 > K^2 > \ldots > K^N$ and $v^1 < v^2 < \ldots < v^N$. As such, no supplier is dominated, and an order could be placed from any of them. Also, we assume that $p > v^N$. It is easy to see that the setting explained above leads to a piecewise linear concave ordering cost structure, such that each supplier corresponds to a particular linear segment. A piecewise concave ordering cost function implies the following important property on the optimal ordering policy: it is always less costly to procure from a single supplier rather than multiple suppliers in any given period [18]. The problem is then to make the supplier selection and to determine replenishment schedule as well as the replenishment quantities so as to minimize the expected total cost.

The generalized (s,S) policy is optimal for the inventory problem with piecewise linear ordering costs [17]. The generalized (s,S) policy extends the traditional (s,S) policy by allowing multiple re-order and order-up-to levels each of which are associated with a particular supplier. By means of these critical levels, supplier selection and replenishment decisions are dynamically made depending upon the observed inventory position. In a similar fashion, we adapt the traditional (R,S) policy for multi-supplier environments. We refer to this new policy as the generalized (R,S) policy. As opposed to the generalized (s,S) policy, the generalized (R,S) policy makes the decisions on supplier selection and replenishment schedule at the very beginning of the planning horizon while determining the replenishment quantities dynamically. In the following sections, we discuss generalized (s,S) and (R,S) policies in detail.

3. The generalized (s,S) policy

In this section, we take a closer look at the inventory problem under consideration, and examine the structure of the optimal control policy. The expected total cost of the inventory system is comprised of ordering, holding and penalty costs. If the inventory position at period t immediately after the delivery is y, then the sum of expected holding and penalty costs to be incurred during that period can be written as

$$L_t(y) = \mathrm{E}\left\{h(y-\xi_t)^+ + p(y-\xi_t)^-\right\} \tag{1}$$

where $x^+ = \max\{0, x\}$, and $x^- = \max\{0, -x\}$.

Then, given an initial inventory position of x units, the optimal expected total cost from period t and onwards can be expressed by means of the dynamic program

$$C_t(x) = \min_{n=1,\ldots,N}\left\{\min_{y \geq x}\left\{K^n\delta(y-x) + v^n(y-x) + L_t(y) + \mathrm{E}C_{t+1}(y-\xi_t)\right\}\right\} \tag{2}$$

where $\delta(x) = 0$ if $x \leq 0$ and 1 otherwise, and the terminal cost function $C_{T+1}(x) = 0$ for all x.

Yu and Benjafaar [17] transformed the one-dimensional dynamic program in (2) into an equivalent n-dimensional one, and exploited its structural properties in order to establish the optimality of the generalized (s,S) policy. The generalized (s,S) policy is a multi-level order-up-to policy characterized by a number of critical parameters in each period t, i.e. $s_t^m < s_t^{m-1} < \ldots < s_t^1 \leq S_t^1 < S_t^2 < \cdots < S_t^m$ for some $m \leq N$ where N is the number of suppliers. The ordering rule for period t is (i) if $x < s_t^m$, then order up to S_t^m, (ii) if $s_t^i \leq x < s_t^{i-1}$, then order up to S_t^{i-1}, and (ii) if $x \geq s_t^1$, then do not order. We refer the reader to Porteus [9,10], Fox et al. [2], Zhang et al. [18], and Yu and Benjafaar [17] for a detailed proof of the optimality of the generalized (s,S) policy.

Although the structure of the optimal policy is known, computing the parameters of the optimal policy is challenging since it requires to solve the continuous state space stochastic dynamic program given above for each and every period within the planning horizon. Here, an option could be to replace the original continuous demand distributions with discrete approximations. Although it is still computationally challenging, this approach enables one to compute the policy parameters by means of conventional methods of finite state space stochastic dynamic programs. In the current study, we employ this approach to compute the parameters of the optimal policy.

4. The generalized (R,S) policy

The (R,S) policy is an order-up-to policy whose essence lies in using a rigid replenishment schedule that is established at the very beginning of the planning horizon while allowing flexibility in the replenishment quantities. The order quantities are dynamically determined at replenishment epochs so as to raise the inventory position to prescribed order-up-to levels. Therefore, the policy specifies the replenishment periods and corresponding order-up-to levels minimizing the expected total cost.

Hereby, we extend the (R,S) policy to multi-supplier inventory systems, and refer to this new policy as the generalized (R,S) policy. If there are multiple suppliers, then the policy should also accommodate a supplier selection decision. The generalized (R,S) policy is therefore an order-up-to policy characterized by a set of