Computers & Operations Research 65 (2016) 174–188

Contents lists available at ScienceDirect

Computers & Operations Research

journal homepage: www.elsevier.com/locate/caor

The impact of hub failure in hub-and-spoke networks: Mathematical formulations and solution techniques

Nader Azizi [a],*, Satyaveer Chauhan [b], Said Salhi [a], Navneet Vidyarthi [b]

[a] *Centre for Logistics and Heuristic Optimisation, Kent Business School, University of Kent, Canterbury, Kent CT27PE, UK*
[b] *John Molson School of Business, Concordia University, Montreal, Quebec, Canada H3G 1M8*

ARTICLE INFO

Available online 27 May 2014

Keywords:
Hub-and-spoke network
Evolutionary algorithm
Hub failure
Logistics
Backup facilities

ABSTRACT

Hub facilities are subject to unpredictable disruptions caused by severe weather condition, natural disasters, labor dispute, and vandalism to cite a few. Disruptions at hubs result in excessive transportation costs and economic losses as customers (demand) initially served by these facilities must now be served by other hubs. In this study, we first present a novel mathematical model that builds hub-and-spoke systems under the risk of hub disruption. In developing the model, we assume that once a hub stops normal operations, the entire demand initially served by this hub is handled by a backup facility. The objective function of the model minimizes the weighted sum of transportation cost in regular situation and the expected transportation cost following a hub failure. We adopted a linearization for the model and present an efficient evolutionary approach with specifically designed operators. We solved a number of small problem instances from the literature using CPLEX for our enhanced mathematical model. The obtained results are also used as a platform for assessing the performance of our proposed meta-heuristic which is then tested on large instances with promising results. We further study and provide results for the relaxed problem in which demand points affected by disruption are allowed to be reallocated to any of the operational hubs in the network.

1. Introduction

The classical hub location problem deals with locating hub facilities and allocating demand to hubs to direct the flow between origin–destination pairs. In the hub location literature, it is common to assume that there is a link between every hub pair, there is no direct path between non-hub nodes, and there is economies of scale for using the inter-hub connections [2]. Depending on how non-hub nodes are allocated to hub facilities, two types of network are constructed namely single and multiple allocations. In the former, all the incoming and outgoing traffic of every node is transferred through a single hub, while in the latter each node in the network can receive and send flow through more than one hub. In this research, we focus on the single allocation p-hub median problem with hub unavailability consideration which we term SApHM-HU.

The hub location problem has various applications in the areas of transportation e.g., air passenger and cargo [5,28,29,35], less-than-truckload freight [11], rail freight [19], urban public transportation and rapid transit [31]. Other applications areas include postal delivery [15,8], express package and cargo delivery [24,39,3], telecommunications [22,7] and supply chains [25]. Hub-and-spoke systems have been the subject of many studies in the past three decades. O'Kelly [32,33] presented the first mathematical model for the single allocation p-hub median problem. Campbell [6] developed a linear integer formulation for the problem. Examples of other formulations that have been proposed in the literature include Ernst and Krishnamoorthy [15], Skorin-Kapov et al. [36], and Ebery [13]. The objective of the p-hub median problem is to determine the location of a predetermined number of facilities (p) and the allocation of the non-hubs to these open hubs such that the total transportation cost is minimized.

Traditional approaches to hub location problem assume that hub facilities are always available. In practice, however, one or more of these facilities may become unavailable from time to time due to, for example, weather conditions and/or natural disasters. To manage hub failure, two strategies are usually adopted in air transportation which include reactive (e.g., canceling, delaying, rescheduling, etc. [18]) and proactive strategies (e.g., investment in reliability improvement of existing facilities). Nevertheless, a disruption at a hub may significantly affects service level and result in excessive transportation cost as customers (demand) initially served by these facilities must now be served by other hubs.

---

* Corresponding author. Tel.: +44 1634 888941.
E-mail address: n.azizi@kent.ac.uk (N. Azizi).

http://dx.doi.org/10.1016/j.cor.2014.05.012
0305-0548/

1.1. Illustrating the impact of hub failure via an example

To evaluate the impact of hub failure on operating cost, we simulate service disruptions in a problem instance with 10 nodes and 3 hubs taken from the U.S. Civil Aeronautics Board which is known as CAB dataset [32]. The dataset is based on the airline passenger interactions between 25 US cities in 1970 and has been frequently used by hub location researchers. Fig. 1 illustrates the optimal network configuration for this problem where inter-hub discount factor i.e., α is 0.2. The total transportation cost of the network presented in Fig. 1 in regular situation is 491.93 units [36].

We assume that once a hub becomes unavailable, the flow initially passing through this facility is rerouted via one of the operating hubs in the network. In Fig. 1, for instance, if hub 6 (Cleveland) is disrupted then the entire flow that uses this hub as the first or the second hub in the path from origin i to destination j is rerouted via either hub 4 (Chicago) or hub 7 (Dallas-Fw). This rerouting strategy is important as in some applications a group of spokes need to be communicated via a single hub to which they are allocated. For instance, in postal service hub facilities are major sorting centres equipped with sorting machinery, optical recognition units, etc. These facilities provide a service to nearby regional offices. In such a system instead of assigning one vehicle between spoke-hub pair, a small fleet will operate for each hub region and each vehicle will visit a subset of cities on their own tours [8,30].

In the simulation, we examine three cases where one of the existing hubs in the network is assumed to be disrupted at a given time. The total network cost corresponding to each case, that includes the following cost elements, is then calculated. The first element of the resulting network cost is the *rerouting* cost of the flow through a backup facility. The rerouted flow initially uses the disrupted hub as either its first or second hub in the path from origin i to destination j. The second element is the *demand loss* cost that measures the cost of not meeting the demand at a disrupted facility (i.e., cost of the flow that either initiates or ends up at the disrupted hub). The third and the final element is the cost of transporting the flow between nodes that are not affected by the hub disruption (*routing* cost). The above three types of cost (i.e., routing, rerouting and demand loss costs) when summed up together make up the *new network cost*.

The total cost of the three new networks corresponding to the above three cases is summarized in Table 1 where "Min" sums the demand loss cost and the smaller of the two routing-rerouting costs associated with each scenario. Each of the two routing-rerouting costs in Table 1 corresponds to the case in which one of the operating hubs in the network is utilized as the backup facility for the disrupted hub. The lower of the two costs associated with the case where the most (economically) attractive rerouting path (i.e., the best backup facility) is utilized to maintain network operations; "Max" represents the network cost when the least attractive backup is utilized to transfer the flow.

Comparison of "Min" network costs for all three scenarios in Table 1 indicates that scenario 1 in which hub 6 is assumed disrupted and hub 4 is utilized as its backup has the lowest cost. The highest cost belongs to the case where hub 4 is assumed disrupted and any of the two other operating hubs in the network (i.e., hub 6 or hub 7) is utilized as the backup facility.

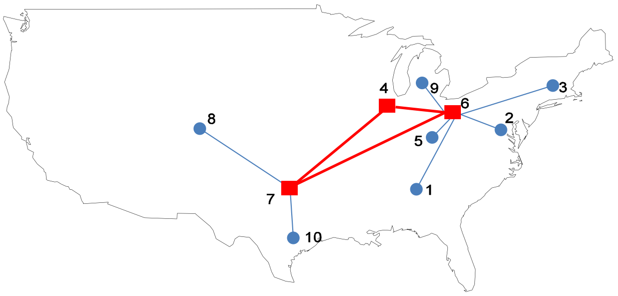
The results presented in Table 1 show that in the event of hub failure, deciding on backup facility largely affects the operating cost. For instance, in Case 1 if hub 7 is utilized as the backup for the assumed disrupted hub 6, the resulting network cost is estimated to be 1149.25 units. However, the network cost significantly reduces (709.41 unit) if hub 4 is used as the backup for hub 6 in the event of hub failure. Our results for the relatively small problem described above further indicates that hub failure causes an excessive cost which on average could increase the regular transportation cost by nearly 89%.

With regard to routing and rerouting costs, our analysis of the results presented in Table 1 shows that the most expensive hub in the network is hub 6. If disrupted, it will impose the largest amount of routing-rerouting cost to the system. This is understandable as more flow is transferred through this hub (i.e., hub 6) in comparison to the other two (hub) facilities in the network (see Fig. 1). One way to guard against such a scenario is to protect such a facility by increasing the level of security which obviously will require extra investment. There are however studies that incorporate these aspects into the modeling. Concerning the demand loss cost, results in Table 1 show that the most expensive facility is hub 4. This hub is the origin/destination of a significant amount of flow which is much higher than that in any of the two other hubs in the network. Therefore, the penalty for not meeting the demand in hub 4 is expected to be relatively high.

Using the data presented in Table 1, the lowest *Expected Transportation Cost* of the network is calculated by multiplying the minimum network costs (under column "Min" in Table 1) and

Table 1
New network costs following a single hub failure.

Disrupted hub	Backup hubs	Routing-rerouting cost	Demand loss cost	New network cost		
				Min	Max	Average
6	4	549.72	159.69	709.41	1149.25	929.33
	7	989.56				
4	6	317.94	540.48	858.43	858.43	858.43
	7	317.94				
7	4	484.80	273.01	757.81	808.92	783.37
	6	535.90				



1 Atlanta	2 Baltimore	3 Boston	**4 Chicago**	5 Cincinnati
6 Cleveland	**7 Dallas-Fw**	8 Denver	9 Detroit	10 Houston

Fig. 1. Optimal solution to a problem with 10 nodes and three hubs (CAB dataset) – single allocation. 🟥 Hub facility.