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An improved formulation for the maximum coverage patrol routing problem

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ABSTRACT

We present an improved formulation for the maximum coverage patrol routing problem (MCPRP). The main goal of the patrol routing problem is to maximize the coverage of critical highway stretches while ensuring the feasibility of routes and considering the availability of resources. By investigating the structural properties of the optimal solution, we formulate a new, improved mixed integer program that can solve real life instances to optimality within seconds, where methods proposed in prior literature fail to find a provably optimal solution within an hour. The improved formulation provides enhanced highway coverage for both randomly generated and real life instances. We show an average increase in coverage of nearly 20% for the randomly generated instances provided in the literature, with a best case increase over 46%. Similarly, for the real life instances, we close the optimality gap within seconds and demonstrate an additional coverage of over 13% in the best case. The improved formulation also allows for testing a number of real life scenarios related to multi-start routes, delayed starts at the beginning of the shifts, and taking a planned break during the shift. Being able to solve these scenarios in short durations help decision and policy makers to better evaluate resource allocation options while serving public.

1. Introduction

Speeding, driving under the influence, and other aggressive driving behaviors are among the leading causes of highway crashes and fatalities. State officials continuously work to discourage such behaviors in several ways, including: (i) increased data-driven enforcement; (ii) technological advances, such as automated enforcement; and (iii) public information and education programs [7]. Data-driven enforcement involves developing strategic countermeasures and operational plans using locally collected data and hot spot information. *Hot spots* are defined as certain combinations of highway stretches and time of day with high frequencies of crashes. The visibility of law enforcement officers at hot spots is known to be one of the key deterrents to aggressive driving. Researchers help government entities in their data-driven enforcement efforts in two main ways. The first area is related to defining hot spots via clustering analysis of historical crash and citation data [2–4,6,11–14]. The second area in data-driven law enforcement, to which this paper contributes, is concerned with the use of predetermined hot spot information in setting effective operational plans that enhance public safety [9,10].

We present here an improved formulation to the maximum covering and patrol routing problem (MCPRP) that was first defined by Keskin et al. [9]. In the MCPRP, certain locations on the highways are "hot" at certain times. The objective of the MCPRP is to maximize coverage of hot spots with a fixed set of state troopers, belonging to a single trooper post. Keskin et al. [9] formulate a mixed integer linear model to solve MCPRP. By demonstrating similarities to the team orienteering problem with time windows [5,16,17], Keskin et al. [9] show that MCPRP is NP-Hard and resort to local- and tabu-search based heuristics.

Our improved formulation retains the original assumptions of the MCPRP formulation [9], including (i) that deterrence is not increased by the simultaneous presence of multiple state trooper vehicles, and (ii) that distinct hot spots at the same physical location (but different times) may be patrolled by different vehicles. In this paper, we first investigate the structural properties of the optimal solution of the MCPRP and prove that an optimal solution must exist in which no hot spot is patrolled by more than one state trooper. This result, together with other formulation strengthening techniques, leads to a significant reduction of the number of variables, and thus of the size of the solution space. Reformulating a problem and improving bounds are common techniques to solve

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"hard" optimization problems in the literature [1,8,15]. With the improved formulation, we solve real life test instances and large randomly generated test instances to optimality within a short time, where Keskin et al. [9] fail to solve those instances to optimality. Moreover, we improve the coverage of the hot spots as much as 46.19% and 13.6% respectively for randomly generated and real life instances. This new model provides the state troopers with an optimal plan that is quickly computable.

Additionally, the new formulation makes it possible to test a number of practical situations for state troopers. These scenarios include (i) letting state troopers start their patrol from their homes as opposed to the state trooper post (multi-depot vs single-depot); (ii) delaying the start time of the patrol due to other duties; and (iii) taking a planned break during the patrol. None of these scenarios would have been easily tested with the original MCPRP model. We computationally test these scenarios over extensive number of cases and propose insights to decision and policy makers regarding better resource allocation.

The remainder of this paper is structured as follows. In Section 2, we present the new model, including necessary assumptions and notation. In Section 3, we discuss computational results based on randomly generated data and real data. In Section 4, we present several real life situations where the new model sheds insights on policy and strategy building. Finally, in Section 5, we provide our conclusions and recommendations for future work.

2. Mathematical model

Our problem definition and assumptions are very similar to that of Keskin et al. [9]. For a given shift, we assume that there are N identified hot spots, where each hot spot $i \in \mathcal{N} = \{1, \ldots, N\}$ has earliest time e_i and latest time l_i for its effective coverage duration, with $e_i < l_i$. The dummy locations 0 and $N+1$ represent the state trooper post at the start and end of the shift, respectively. Without loss of generality, we assume that the shift begins at time 0. The objective of the patrol routing problem is to maximize the deterrence effect by finding the best patrol route and determining time spent at hot spots for each state trooper car $k \in \mathcal{K}$. We define our additional notation in Table 1.

The problem setting includes the following assumptions and restrictions, as in Keskin et al. [9]:

1. Each state trooper division can be solved separately.
2. The model horizon is a single shift for one day.
3. State troopers start and end their shift at the state trooper post. Shift duration is enforced by the parameters $e_0 = 0$ and $l_{N+1} = T$.
4. All of the state trooper cars ($k \in \mathcal{K}$) are identical.
5. A state trooper can arrive at hot spot i prior to its start time e_i, but the deterrence effect is only calculated starting at e_i, i.e., only after the hot spot is "hot."
6. Having multiple state troopers in the same hot spot at the same time would provide the same deterrence effect as a single trooper.
7. Travel speed is a constant and is set to 60 miles/hour in the numerical experiment.
8. Trooper cars are in continuous service during the shift. (Meals, refueling etc. are assumed to take place before or after the shift.)

Before introducing the improved formulation for patrol routing problem (IPRP), we present the original patrol routing formulation Keskin et al. [9] for the sake of completeness:

$$\text{Maximize} \sum_{i \in \mathcal{N}} \sum_{k \in \mathcal{K}} (f_{ik} - s_{ik}) \qquad \text{(MCPRP)}$$

Subject to:

$$f_{ik} + t_{ij} - s_{jk} \le (1 - x_{ijk}) M_{ij}, \quad \forall k \in \mathcal{K} \quad \text{and} \quad \forall (i,j) \in \mathcal{E} \qquad (1)$$

$$e_i \sum_{j \in \Delta^+(i)} x_{ijk} \le s_{ik}, \quad \forall k \in \mathcal{K} \quad \text{and} \quad \forall i \in \mathcal{V} \qquad (2)$$

$$l_i \sum_{j \in \Delta^+(i)} x_{ijk} \ge f_{ik}, \quad \forall k \in \mathcal{K} \quad \text{and} \quad \forall i \in \mathcal{V} \qquad (3)$$

$$s_{ik} \le f_{ik}, \quad \forall k \in \mathcal{K} \quad \text{and} \quad \forall i \in \mathcal{V} \qquad (4)$$

$$\sum_{j \in \Delta^+(0)} x_{0jk} = 1, \quad \forall k \in \mathcal{K} \qquad (5)$$

$$\sum_{i \in \Delta^-(N+1)} x_{i,N+1,k} = 1, \quad \forall k \in \mathcal{K} \qquad (6)$$

$$\sum_{i \in \Delta^-(j)} x_{ijk} = \sum_{i \in \Delta^+(j)} x_{jik}, \quad \forall k \in \mathcal{K} \quad \text{and} \quad \forall j \in \mathcal{N} \qquad (7)$$

$$\sum_{j \in \Delta^+(i)} x_{ijk} = y_{ik}, \quad \forall k \in \mathcal{K} \quad \text{and} \quad \forall i \in \mathcal{N} \qquad (8)$$

$$y_{0,k} = y_{N+1,k} = 1, \quad \forall k \in \mathcal{K} \qquad (9)$$

$$u_{ikg} + u_{igk} \le y_{ik}, \quad \forall i \in \mathcal{V} \quad \text{and} \quad k, g \in \mathcal{K}, \ g > k \qquad (10)$$

$$u_{ikg} + u_{igk} \le y_{ig}, \quad \forall i \in \mathcal{V} \quad \text{and} \quad k, g \in \mathcal{K}, \ g > k \qquad (11)$$

$$u_{ikg} + u_{igk} \ge y_{ik} + y_{ig} - 1, \quad \forall i \in \mathcal{V} \quad \text{and} \quad k, g \in \mathcal{K}, g > k \qquad (12)$$

$$f_{ik} - s_{ig} - M(1 - u_{ikg}) \le 0, \quad \forall i \in \mathcal{V} \quad \text{and} \quad k, g \in \mathcal{K}, \ g > k \qquad (13)$$

$$f_{ig} - s_{ik} - M(1 - u_{igk}) \le 0, \quad \forall i \in \mathcal{V} \quad \text{and} \quad k, g \in \mathcal{K}, \ g > k \qquad (14)$$

Table 1
Notation.

Problem parameters:	
$\mathcal{V}$	Set of hot spots and state trooper post, $\mathcal{V} = \mathcal{N} \bigcup \{0, N+1\}$.
$\mathcal{E}$	Set of arcs, $\mathcal{E} = \{(i,j) : i,j \in \mathcal{V}, i \ne j\}$.
$\Delta^+(i)$	Set of hot spots reachable from $i \in \mathcal{V}$ within their time window, $\Delta^+(i) = \{j \in \mathcal{V} : (i,j) \in \mathcal{E}, e_i + t_{ij} \le l_j\}$ (see below for the definitions of e, t and l).
$\Delta^-(i)$:	Set of hot spots from which $i \in \mathcal{V}$ is reachable, $\Delta^-(i) = \{j \in \mathcal{V} : (j,i) \in \mathcal{E}, e_j + t_{ji} \le l_i\}$.
t_{ij} :	Shortest travel time from $i \in \mathcal{V}$ to $j \in \mathcal{V}$.
T:	End of shift.
Decision variables:	
x_{ijk} :	1, if state trooper car $k \in \mathcal{K}$ travels from hot spot i to j, $(i,j) \in \mathcal{E}$; 0, otherwise.
$s_{ik} \ge 0$:	Start of patrol at hot spot $i \in \mathcal{V}$ by state trooper $k \in \mathcal{K}$.
$f_{ik} \ge 0$:	End of patrol at hot spot $i \in \mathcal{V}$ by state trooper $k \in \mathcal{K}$.
y_{ik} :	1, if state trooper $k \in \mathcal{K}$ patrols hot spot $i \in \mathcal{V}$; 0, otherwise.