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Linear programming models for a stochastic dynamic capacitated lot sizing problem

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ABSTRACT

In this paper the stochastic dynamic lot sizing problem with multiple items and limited capacity under two types of fill rate constraints is considered. It is assumed that according to the static-uncertainty strategy of Bookbinder and Tan [2], the production periods as well as the lot sizes are fixed in advance for the entire planning horizon and are executed regardless of the realisation of the demands. We propose linear programming models, where the non-linear functions of the expected backorders and the expected inventory on hand are approximated by piecewise linear functions. The resulting models are solved with a variant of the Fix-and-Optimize heuristic. The results are compared with those of the column generation heuristic proposed by Tempelmeier [14].

1. Introduction

Lot sizing problems occur in industrial practice, when a production process can only start after a setup of the required resources with associated setup time and/or setup costs has been completed. In the literature, numerous lot sizing approaches for different production situations have been proposed. Particularly relevant for operational production planning in manufacturing companies are dynamic lot sizing models, as they consider demands and orders of varying sizes associated with specific due dates.

The majority of the lot sizing literature focusses on the situation when all data are deterministically known in advance. Industrial planning practice usually applies a forecasting procedure that provides a deterministic time series of the expected future demands. Uncertainty is taken into consideration by reserving a fixed amount of inventory as safety stock. The amount of this reserve stock is usually computed by simple rules of thumb, e.g. the standard deviation of the demand during the risk period is multiplied by a quantile of the standard normal distribution. In this way, it is usually not possible to meet a target service level. In addition, the effect of the lot sizes on the risk hedging is not taken into consideration. For example, with large lot sizes it may be optimal to provide no safety stock at all.

In this paper, we consider the stochastic dynamic multi-item capacitated lot sizing problem (SCLSP), which is the stochastic counterpart of the well-known deterministic dynamic multi-item capacitated lot sizing problem (CLSP). The problem can be described as follows. We consider a single resource, which is used to produce K $(k = 1, 2, \ldots, K)$ items with dynamic random period demands D_{kt} over a planning horizon of T $(t = 1, 2, \ldots, T)$ periods. For product k, the demands D_{kt} are random variables with forecasted period-specific expectation $E\{D_{kt}\}$ and variance $V\{D_{kt}\}$. The period capacities of the resource are b_t $(t = 1, 2, \ldots, T)$.

We assume that the "static-uncertainty strategy" according to [2] is in place, which means that at the beginning of the planning horizon the complete production plan is fixed, including the timing and the size of production quantities. Unlike the "dynamic-uncertainty strategy" and the "static-dynamic-uncertainty strategy" which result in random lot sizes, this strategy has the advantage that it is possible to construct a production plan that respects limited capacities with certainty.

This problem definition reflects the scenario that can be observed in MRP planning environments and in so-called Advanced Planning Systems. With given setup s_k and holding costs h_k $(k = 1, 2, \ldots, K)$, we seek to determine production quantities to satisfy the time-varying random period demands so as to minimize the sum of setup and holding costs. Inventory holding costs are charged on the inventory at the end of each period. As backlog costs are usually difficult if not impossible to quantify, we assume that a β service level (fill rate) is used as a performance criterion.

The β service level relates the total amount backordered to the total demand observed during a given time span, whereby the

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backorder in period t, B_t, depends on the demand observed in period t, D_t, and the available inventory at the beginning of period t. In stochastic inventory systems, under stationary conditions, usually the long-term average fill rate is considered. However, the β service level can also be calculated for any finite number of periods (see [1,4,17]). Let $Y^{(t)}$ be the cumulated demand and let $B^{(t)}$ be the cumulated backorders from period 1 to t. Then the fill rate calculated w.r.t. periods 1 to t, β_t, is defined as $\beta_t = 1 - B^{(t)}/Y^{(t)}$.

As an alternative to β_t, the cycle service level β_c relates the backorders within a replenishment cycle to the demand that occurs in that cycle. This criterion can only be calculated if the cycle length is known. If two production orders are released in, for example, periods 3 and 6, then the coverage time of the first order runs from period 3 to 5. For the calculation of the β_c service level the backorders, which newly occur in periods 3 to 5 are added and related to the demands of these three periods. A β_c service level constraint is more restrictive than a β constraint, because it requires the achievement of the target in each and every cycle. A bad performance in one cycle cannot be compensated by a good performance in a different cycle, which would be possible with the β criterion.

It can be shown that β_c can be expressed in terms of cumulated backorders and cumulated demands. If two consecutive order cycles $(i-1)$ and i end in periods τ_{i-1} and τ_i, respectively, then $\beta_c(\tau_i)$ is defined as

$$\beta_c(\tau_i) = 1 - \frac{B^{(\tau_i)} - B^{(\tau_{i-1})}}{Y^{(\tau_i)} - Y^{(\tau_{i-1})}}, \tag{1}$$

where the numerator describes the net backorders, which newly occurred in cycle i and the denominator is the corresponding demand. There is an interesting relation between β_{τ_i}, $\beta_{\tau_{i-1}}$, and $\beta_c(\tau_i)$, which is useful for the formulation of a lot sizing model. At the end of period τ_i, the finite period service level is defined as

$$\beta_{\tau_i} = 1 - \frac{B^{(\tau_i)}}{Y^{(\tau_i)}}. \tag{2}$$

Similarly, at the end of period τ_{i-1}, we have

$$\beta_{\tau_{i-1}} = 1 - \frac{B^{(\tau_{i-1})}}{Y^{(\tau_{i-1})}}. \tag{3}$$

Now, assume that the production quantities are set such that at the end of each production cycle the finite period service levels are equal: $\beta_{\tau_i} = \beta_{\tau_{i-1}} = X$.

Then

$$(1-X) \cdot Y^{(\tau_i)} = B^{(\tau_i)} \tag{4}$$

and

$$(1-X) \cdot Y^{(\tau_{i-1})} = B^{(\tau_{i-1})} \tag{5}$$

Taking the difference between (4) and (5), we obtain

$$(1-X) \cdot \left(Y^{(\tau_i)} - Y^{(\tau_{i-1})}\right) = \left(B^{(\tau_i)} - B^{(\tau_{i-1})}\right) \tag{6}$$

or

$$1 - X = \frac{B^{(\tau_i)} - B^{(\tau_{i-1})}}{Y^{(\tau_i)} - Y^{(\tau_{i-1})}} = 1 - \beta_c(\tau_i). \tag{7}$$

Hence, if lot sizes are set such that $\beta_{\tau_i} = \beta_{\tau_{i-1}}$, then $\beta_c(\tau_i) = \beta_{\tau_i} = \beta_{\tau_{i-1}}$. As a consequence, it is possible to meet a β_c service level target through the introduction of surrogate β_t service level constraints in a lot sizing model. This is what we are proposing in this paper. Thereby, in the constraints of the model, we quantify the actual service level by the ratio of the expected values of the backorders and the demands, which is only exact for an infinite time horizon. As for a limited time horizon t the relation $(1 - E\{B^t/Y^t\}) \geq (1 - E\{B\}/E\{Y\})$ holds [1], this is a conservative approximation which ensures that the target set by the management will be met.

The rest of this paper is organized as follows. In Section 2 the relevant literature is reviewed. In Section 3 we describe the problem in detail. The model formulations are presented in Section 4. Following, the solution approach is presented in Section 5. The results of a numerical experiment are reported in Section 6. Finally, Section 7 contains some concluding remarks.

2. Literature

In the literature, we observe a rapidly increasing amount of papers on stochastic lot sizing problems. However, only a limited number of researchers have considered dynamic capacitated lot sizing problems with random demand and service level constraints. Reviews of stochastic lot sizing problems which deal with multiple items produced on a single resource with limited capacity are presented by Sox et al. and Winands et al. [10,18] and in chapter E in [15]. Sox and Muckstadt [11] solve a variant of the stochastic dynamic CLSP, where item- and period-specific backorder costs as well as extendible production capacities are considered. The authors propose a Lagrangean heuristic to solve the resulting non-linear integer programming problem that is repeatedly applied in a dynamic planning environment. Brandimarte [3] considers the stochastic CLSP where the uncertainty of the demand is represented by a scenario tree. In this case, the period demands are modeled as discrete random variables. The evolution of demand over time is depicted with a directed layered tree, where each layer corresponds to a planning period and the nodes are linked to realizations of the discrete stochastic demand process. The resulting large-scale deterministic MIP model is then solved with a commercially available solver using rolling schedules with lot sizing windows. As demonstrated by Brandimarte [3], the scenario-based approach suffers from a dramatically increasing complexity, if the number of periods and/or the number of possible outcomes of the period demands are increased. In addition, currently there are no scenario-based models available which could account for product-specific fill rate constraints. Tempelmeier and Herpers [16] propose a formulation of the dynamic capacitated lot sizing problem under random demand, when the performance is measured in terms of a fill rate per cycle β_c. They propose the ABC$_\beta$ heuristic which is an extension of the A/B/C heuristic proposed for the solution of the deterministic CLSP by Maes and van Wassenhove [8]. A different solution approach which outperforms the ABC$_\beta$ heuristic is presented in [14]. The author proposes a heuristic solution procedure that combines column generation and the ABC$_\beta$ heuristic. Helber et al. [7] consider a formulation of the stochastic CLSP with a so-called δ service level constraint which takes the duration of stockouts into consideration. They develop a model where the nonlinear functions of expected backlog and expected inventory on hand are approximated by use of piecewise linear segments. The resulting piecewise linear model is solved with a MIP-based heuristic.

In the current paper we extend the linearized MIP model of [7] through the introduction of β service level constraints, which are more common in industrial practice. In addition, we extend the basic model through the introduction of setup carry-overs, which lead to a better representation of the dynamic lot sizing problem. We apply a Fix&Optimize heuristic to solve a large number of test problem instances and compare the results with the results of the column generation heuristic proposed by Tempelmeier [14].

3. Problem statement

We consider K products that are produced to stock on a single resource with limited period capacities b_t $(t = 1, 2, \ldots, T)$. The planning situation is completely identical with the classical CLSP with