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Impatient customer queue with Bernoulli schedule vacation interruption

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ABSTRACT

This paper deals with an infinite buffer *M/M/1* queue with working vacations and Bernoulli schedule vacation interruption wherein the customers balk with a probability. Whenever the system becomes empty, the server takes a working vacation during which service is provided with a lower rate and if there are customers at a service completion instant, vacation is interrupted and the server resumes a normal working period with probability q or continues the vacation with probability $1-q$. The service times during working vacation and vacation times are assumed to be exponentially distributed. During a working vacation customers may renege due to impatience. The closed form expressions of the steady-state probabilities and the performance measures of the model are obtained using generating functions. Various numerical results are presented to show the effect of the model parameters on the system performance measures.

1. Introduction

Queueing models with server vacations have been well studied in the past three decades and successfully applied in many areas such as computer and communication systems, manufacturing and production systems, service systems, and telecommunication systems. Excellent surveys on vacation models have been reported in Doshi [1], Tian and Zhang [2], Ke et al. [3], and so on. Working vacation (WV) is one kind of vacation policy under which the server provides service at a lower speed during the vacation period rather than stopping service completely. It was introduced by Servi and Finn [4] in an *M/M/1* queueing system.

In this paper, we analyze a queue for two WV policies, the multiple working vacations (MWV) and single working vacation (SWV). In MWV the server takes vacations until it finds at least one customer waiting in the system at a vacation completion instant. Under a SWV policy, the server takes exactly one WV at the end of each busy period. As this SWV ends, the server either serves the customers, if any, at a normal service rate or stays idle. Wu and Takagi [5] generalized Servi and Finn's *M/M/1*/WV queue to an *M/G/1*/WV queue. Baba [6] extended [4] to a renewal input *GI/M/1* queue with WV and derived the steady-state system length distributions at pre-arrival and arbitrary epochs. Tian et al. [7] studied an *M/M/1* queue with SWV using matrix geometric method. Li and Tian [8] generalized this study to *GI/M/1* queue with SWV and obtained the queue length distributions at pre-arrival and arbitrary epochs using matrix-geometric approach. During a WV period, the server can interrupt the vacation at a service completion instant if there are customers in the system. This is known as WV with vacation interruption (VI). For the vacation interruption models, Li and Tian [9] first introduced it in an *M/M/1* queue. Li et al. [10] generalized this study to *GI/M/1* queue with WV and VI using matrix-analytic method. Adopting the method of supplementary variable, Zhang and Hou [11] investigated the *M/G/1* queue with WV and VI.

Under the Bernoulli schedule VI, the server is allowed to interrupt the vacation with probability q, if there are customers in the queue at a service completion epoch during WV, or continues the vacation with probability $1-q$. Zhang and Shi [12] first studied an *M/M/1* queue with VI under the Bernoulli rule. Li et al. [13] studied a *GI/M/1* queue with start-up period, SWV and Bernoulli schedule vacation interruption using embedded Markov chain technique. The *GI/M/1* queue with Bernoulli schedule controlled vacation and vacation interruption has been analyzed by Li et al. [14] using the matrix analytic method. Shan and Liu [15] have analyzed an *M/G/1* queue with SWV and VI under Bernoulli schedule using supplementary variable technique and matrix geometric method.

In real life, many queueing situations arise wherein the customers tend to be discouraged by a long queue. As a result, the customers either decide not to join the queue (i.e., balk) or depart after joining the queue without getting served due to impatience (i.e., renege). These phenomena are observed in situations involving impatient

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telephone switchboard customers, hospital emergency rooms handling critical patients, inventory systems that store perishable goods, and so on. Altman and Yechaili [16] have analyzed impatient customer queues with server vacations. Infinite server queues with systems' additional task and impatient customers have been considered in [17]. Yue et al. [18] studied customers' impatience in an *M/M/1*/MWV queue where the customers impatient is due to WV. A renewal input finite buffer MWV queue with balking has been studied by Vijaya Laxmi and Jyothsna [19]. Recently, Selvaraju and Cosmika [20] analyzed an *M/M/1* impatient customer queue with single and multiple working vacations.

This paper is an extension of the earlier works [18,20]. In [18], the authors have considered an *M/M/1* queue with MWV and reneging of customers. Selvaraju and Cosmika [20] extended this work by including SWV. In this paper, we further generalize [20] by incorporating the features of customers balking and Bernoulli schedule vacation interruption. Using probability generating functions, the explicit expressions of the steady-state probabilities are obtained. Various performance measures such as the average system length, average balking rate, and average reneging rate are discussed. Stochastic decomposition of average system length and average sojourn time is also carried out. With these performance measures, we demonstrate the parameter effect on the performance indices of the system. Finally, several queueing models available in the literature can be deduced as special cases of our model.

The rest of the paper is organized as follows. Section 2 presents a brief description of the model. The explicit expressions of the steady-state probabilities are obtained in Section 3. Various performance measures of the model and stochastic decomposition properties are discussed in Section 4. A variety of numerical results are presented in Section 5 followed by conclusions in Section 6.

2. Model description

Let us consider an infinite buffer single server queueing system with WV and Bernoulli schedule VI. Customers arrive according to a Poison process with rate λ. On arrival, a customer either decides to join the queue with probability b or balks with probability $\overline{b}$, where for any real number $x \in [0,1]$, we denote $\overline{x} = 1-x$. However, when the system is empty, an arriving customer joins the system with probability 1. Customers are served according to FCFS service discipline and the service times during a normal working period are assumed to be exponentially distributed with parameter μ.

The server commences a WV of random length at the instants the system becomes empty. If customers arrive during a WV, they are served at a rate which is different (generally lower) from the normal service rate, μ. The vacation times and service times during WV are also assumed to be exponentially distributed with parameter ϕ and η, respectively. The server is assumed to interrupt the vacation under the Bernoulli rule, i.e., at a service completion instant during WV if there are customers in the system, the server may interrupt the vacation and switch to normal working period with probability q or continues the vacation with probability $\overline{q}$. We present a combined approach to analyze single and multiple working vacation models with VI together and for that we define an indicator function δ such that for $\delta = 1$, one can get the results for SWV and $\delta = 0$ gives the results for MWV model.

An arriving customer who joins the system and finds the server in WV, activates an impatient timer T, which is exponential with rate α. If the server is available during WV before the time T expires, the customer is served with rate η. If the WV finishes before the impatient timer expires, the server switches to normal working period and the customer is served with rate μ. If the impatient timer expires and the customer's service has not begun or has not been completed while the server is still in WV, the customer abandons the system and never returns. Since the arrival and departure of an impatient customer without service are independent, the average reneging rate of a customer is given by $n\alpha$, where n denotes the number of customers in the system.

3. Steady-state probabilities

At time t, let $L(t)$ denote the number of customers in the system and $J(t)$ be the status of the server, which is defined as $J(t)=0$ when the server is in WV and $J(t)=1$ when the server is in normal working period. The process $\{(J(t), L(t)); t \geq 0\}$ defines a continuous-time Markov process with state space $\Phi = \{(j,n): j=0, n \geq 0; j=1, n \geq 1-\delta\}$.

Let $\pi_{j,n} = \lim_{t\to\infty} P\{J(t)=j, L(t)=n\}$, $j = 0$, $n \geq 0$; $j = 1$, $n \geq 1-\delta$ denote the steady-state probabilities of the process $\{(J(t), L(t)); t \geq 0\}$. Under the stability condition, $\lambda b < \mu$, the set of balance equations is given as follows:

$$(\lambda+\delta\phi)\pi_{0,0} = (\eta+\alpha)\pi_{0,1} + \mu\pi_{1,1}, \tag{1}$$

$$(\lambda b+\phi+\eta+\alpha)\pi_{0,1} = \lambda\pi_{0,0} + (2\alpha+\overline{q}\eta)\pi_{0,2}, \tag{2}$$

$$(\lambda b+\phi+\eta+n\alpha)\pi_{0,n} = \lambda b\pi_{0,n-1} + ((n+1)\alpha+\overline{q}\eta)\pi_{0,n+1}, n \geq 2, \tag{3}$$

$$\lambda\pi_{1,0} = \delta\phi\pi_{0,0}, \tag{4}$$

$$(\lambda b+\mu)\pi_{1,1} = \lambda\pi_{1,0} + \mu\pi_{1,2} + \phi\pi_{0,1} + q\eta\pi_{0,2}, \tag{5}$$

$$(\lambda b+\mu)\pi_{1,n} = \lambda b\pi_{1,n-1} + \mu\pi_{1,n+1} + \phi\pi_{0,n} + q\eta\pi_{0,n+1}, n \geq 2. \tag{6}$$

The steady-state probabilities are obtained by solving the above system of Eqs. (1)–(6) using probability generating functions (pgf). Define $G_0(z)$ and $G_1(z)$ as

$$G_0(z) = \sum_{n=0}^{\infty} z^n \pi_{0,n}, \quad G_1(z) = \delta\pi_{1,0} + \sum_{n=1}^{\infty} z^n \pi_{1,n},$$

with $G_0(1)+G_1(1)=1$ and $G_0'(z) = (d/dz)G_0(z) = \sum_{n=1}^{\infty} nz^{n-1}\pi_{0,n}$.

Multiplying Eqs. (1)–(3) by $1, z, z^n$, respectively, summing over n and rearranging the terms, we get

$$\alpha z(1-z)G_0'(z) + \left[\lambda bz^2 - (\lambda b+\phi+\eta)z + \overline{q}\eta\right]G_0(z) + Az - Bz(1-z) - C(1-z) = 0, \tag{7}$$

where $A = ((1-\delta)\phi+q\eta)\pi_{0,0} + \mu\pi_{1,1} + q\eta\pi_{0,1}$, $B = \lambda\overline{b}\pi_{0,0}$ and $C = \overline{q}\eta\pi_{0,0}$.

In a similar manner, from Eqs. (4)–(6), we have

$$\begin{aligned}(\lambda bz-\mu)(1-z)G_1(z) &= (\phi z+q\eta)G_0(z) - \left(\mu\pi_{1,1} + (1-\delta)\phi\pi_{0,0} + q\eta\pi_{0,1}\right)z - q\eta\pi_{0,0} \\ &\quad + \delta((\lambda bz-\mu)(1-z) - \lambda z(1-z))\pi_{1,0}.\end{aligned} \tag{8}$$

Remark 1. Letting $\eta = 0$ and $b=1$, Eq. (7) reduces to

$$\alpha(1-z)G_0'(z) = [\lambda(1-z)+\phi]G_0(z) - (\phi(1-\delta)\pi_{0,0} + \mu\pi_{1,1}),$$

which agrees with Altman and Yechiali [16] (for $\delta = 0$, see Eq. (2.4), pg. 263 and for $\delta = 1$, see Eq. (5.3), pg. 274).

Remark 2. Taking $\alpha = q = \delta = 0$ and $b=1$ and rearranging the terms reduces Eq. (7) to

$$G_0(z) = \frac{\eta(1-z)\pi_{0,0} - (\phi\pi_{0,0} + \mu\pi_{1,1})z}{\lambda z^2 - (\lambda+\phi+\eta)z + \eta},$$

which matches with Servi and Finn [4] (see Eq. (A.3) in Appendix A, pg. 49).

Now, we derive the steady-state probabilities by solving Eqs. (7) and (8), following the method used in Yue et al. [18]. Eq. (7) can be written as

$$G_0'(z) + \left[-\frac{\lambda b}{\alpha} - \frac{\phi+\eta}{\alpha(1-z)} + \frac{\overline{q}\eta}{\alpha z(1-z)}\right]G_0(z) + \frac{A}{\alpha(1-z)} - \frac{B}{\alpha} - \frac{C}{\alpha z} = 0$$