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Dynamic design of sales territories

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ABSTRACT

We introduce the Multiple Traveling Salesmen and Districting Problem with Multi-periods and Multi-depots. In this problem, the compactness of the subdistricts, the dissimilarity measure of districts and an equity measure of salesmen profit are considered as part of the objective function, and the salesman travel cost on each subdistrict is approximated by the Beardwood–Halton–Hammersley formula. An adaptive large neighbourhood search metaheuristic is developed for the problem. It was tested on modified Solomon and Gehring & Homberger instances. Computational results confirm the effectiveness of the proposed metaheuristic.

1. Introduction

The problem considered in this paper is the Multiple Traveling Salesman and Districting Problem with Multi-Periods and Multi-Depots, where the customers of a sales territory dynamically evolve over the periods of a planning horizon. The problem consists of designing districts and subdistricts for a multiple traveling salesman problem with dynamic customers over several periods. Each salesman services all customers of his district over the planning horizon but performs a single route in a subdistrict in each period. The customers on the territory vary dynamically over the planning horizon. A proportion η of the customers of the previous period leave the territory, and a proportion ψ of new customers enter it. Typically the number of customers of a territory will tend to increase over time. However, all information on customers, which includes their number and locations, is available at the beginning of each period. There also exist several depots on the territory at reasonable locations. The number of depots and their locations are static over time. Fig. 1 depicts a territory partitioned into districts and subdistricts, with depots, periods and salesmen's routes.

The problem is defined on an undirected graph $G=(V,E,P)$, where $P=\{1,\dots,T\}$ is the set of periods, $V=D\cup V^1\cup\dots\cup V^T$ is the vertex set, D is the depot set at which the salesmen are located, V^t is the set of customers at period t, and $E=\{(v_i,v_j):v_i,v_j\in V, i<j\}$ is the edge set. A symmetric matrix of Euclidean travel times, equal to travel costs, is defined on E. The problem consists of designing several contiguous districts served in each period and subdistricts served in each working day such that (1) all customers within the same district are served by the same salesman, (2) each customer is visited once by one salesman, (3) a service time s is incurred when visiting a customer, (4) each salesman route has a normal duration limit h, but overtime is paid at rate θ if its duration exceeds h, and (5) an objective function combining salesman cost (number of districts), a subdistrict compactness measure, a district partition dissimilarity measure and a salesmen profit equity measure is minimised.

Several companies, such as Coca-Cola, DHL and FedEx, face this problem. They need to segment or partition their customers into clusters or territories in order to efficiently handle marketing and distribution decisions over different periods, and the customer base is not static. In such contexts, it is desirable to consistently assign almost the same customers to each salesman, to create relatively stable districts, and to design equitable subdistricts in terms of workload.

There exists a rich literature on districting. Most of it deals with deterministic problems. The relevant papers include the drawing of political districts [28,5,6,21] the design of school districts [14], the construction of police districts [10], districting for home-care services [4], the alignment of commercial territories [37,12,20,31,26], and the solution of location-districting problems [29,7]. Research on stochastic districting problems has mostly been conducted in the context of vehicle routing. Haugland et al. [18] have considered the problem of designing districts for vehicle routing problems with stochastic demands. The demands are assumed to be uncertain at the time when the districts are designed, and these are revealed only after the districting decisions are determined. A tabu search heuristic was provided for the problem. Lei et al. [24] proposed a vehicle routing and districting problem with stochastic customers. The problem was modeled and solved as a two-stage stochastic program in which the districting

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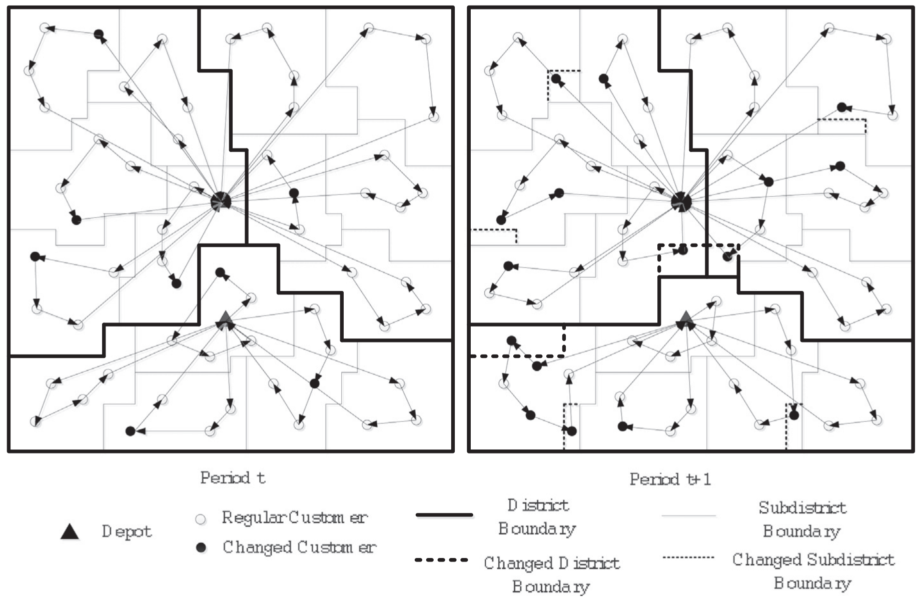

Fig. 1. An example of a territory partitioned into districts and subdistricts, with two depots and salesmen's routes over two periods.

decisions are made in the first stage and the Beardwood–Halton–Hammersley formula was used to approximate the expected routing cost of each district in the second stage. A large neighbourhood search metaheuristic was also developed for the problem. Carlsson and Delage [9] introduced a robust framework for distributing the load of a vehicle routing problem over a fleet of vehicles when the location of demand points and their distribution are not known with certainty. Carlsson [8] has studied an uncapacitated stochastic vehicle routing problem in which vehicle depot locations are fixed and customer locations in a service region are unknown, but are assumed to be independent and identically distributed from a given probability density function.

To the best of our knowledge, this paper is the first to consider dynamic customers in the context of a joint multiple traveling salesmen and districting problem with multi-periods and multi-depots. We have considered the salesman cost, the subdistrict compactness measure, a district partition dissimilarity measure and a salesman profit equity measure in the objective function. Instead of explicitly determining the salesman routes, we approximate their cost by means of the Beardwood–Halton–Hammersley theorem [3]. We integrate this approximation within a large neighbourhood search metaheuristic for the districting phase.

The remainder of the paper is organised as follows. The mathematical model is presented in Section 2. An adaptive large neighbourhood search metaheuristic for the problem is described in Section 3, followed by computational experiments in Section 4, and by conclusions in Section 5.

2. Mathematical model

We introduce the following additional notation: $V^t=\{1,\dots,n^t\}$ is the set of the customers on the territory at period t, where n^t is the associated number of the customers; $D=\{D_1,\dots,D_Z\}$ is the set of the depots, and D_k^t is the depot assigned to district k at period t; m^t is the number of districts at period t; V_k^t is the customer set which are located in the district k at period t, $V_{k,i}^t$ is the customer set which are located in the subdistrict i of district k at period t, and n_k^t is the number of the customers in district k at period t; a is the unit revenue generated by serving a customer; h is the duration limit of a route; $d_{k,i}^t$ is the distance between D_k^t and the customer of $V_{k,i}^t$ closest to D_k^t.

We assume that a period lasts several weeks with a maximum number of working days each week (e.g., from Monday to Friday). The problem is modeled as follows. For period t, the solution is a decomposition of V^t into m^t districts, and partition of district V_k^t into s_k^t subdistricts, each of which corresponding to a salesman tour on a working day. A feasible district and subdistrict plan $x=\left\{V_1^t\{V_{1,1}^t,\dots,V_{1,s_1^t}^t\},\dots,V_{m^t}^t\{V_{m^t,1}^t,\dots,V_{m^t,s_{m^t}^t}^t\}\right\}$ must satisfy following constraints: (1) $\forall D_k^t, D_k^t\in D$; (2) $\{V_1^t,\dots,V_{m^t}^t\}$ is a partition of V^t; (3) $\{V_{k,1}^t,\dots,V_{k,s_k^t}^t\}$ is a partition of district V_k^t.

After the design of districts and subdistricts, the closest depot $D_k^t(D_k^t\in D)$ to the district V_k^t is assigned to the district, and the cost of the salesman tour on $\{D_k^t\}\cup V_{k,i}^t$ is computed for each subdistrict $V_{k,i}^t$. The workload of a subdistrict $V_{k,i}^t$ is approximated as the length of an optimal traveling salesman problem tour over $V_{k,i}^t$, plus twice the distance $d_{k,i}^t$ between D_k^t and the customer of $V_{k,i}^t$ closest to D_k^t. The number m^t of designed districts at period t is a decision variable.

The objective of the model is

$$\min_x F(x)=\sum_{t=1}^{T}\left(\alpha_m m^t+\alpha_{comp}F_{comp}^t(x)+\alpha_{dissim}F_{dissim}^t(x)+\alpha_{equ}F_{equ}^t(x)\right), \tag{1}$$

where x denotes a feasible solution. The objective function minimises the sum over m^t of districts, of the compactness measure $F_{comp}^t(x)$ of the subdistricts, the dissimilarity measure $F_{dissim}^t(x)$ of the district partition, and of the equity measure $F_{equ}^t(x)$ of the salesmen over all periods, weighted by the positive user-defined parameter α_m, α_{comp}, α_{dissim} and α_{equ}. The computation of $F_{comp}^t(x)$, $F_{dissim}^t(x)$ and $F_{equ}^t(x)$ is detailed in Sections 2.1, 2.2 and 2.3 respectively.

2.1. Compactness measure of the subdistricts

As in Bozkaya et al. [6], we use the following formula to measure the compactness of a subdistrict:

$$F_{comp}^t(x)=\left(\sum_{k=1}^{m^t}\sum_{i=1}^{s_k^t}B_{k,i}^t(x)-B^t\right)\Bigg/\left(2B^t\sum_{k=1}^{m^t}s_k^t\right), \tag{2}$$

where $B_{k,i}^t(x)$ and B^t are respectively the perimeters of subdistrict $V_{k,i}^t$ and of the entire territory at period t in solution x, s_k^t