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Multi-product valid inequalities for the discrete lot-sizing and scheduling problem

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ABSTRACT

We consider a problem arising in the context of industrial production planning, namely the multi-product discrete lot-sizing and scheduling problem with sequence-dependent changeover costs. We aim at developing an exact solution approach based on a Cut & Branch procedure for this combinatorial optimization problem. To achieve this, we propose a new family of multi-product valid inequalities which corresponds to taking into account the conflicts between different products simultaneously requiring production on the resource. We then present both an exact and a heuristic separation algorithm which form the basis of a cutting-plane generation algorithm. We finally discuss computational results which confirm the practical usefulness of the proposed inequalities at strengthening the MILP formulation and at reducing the overall computation time.

1. Introduction

We consider an optimization problem arising in the context of industrial production planning, namely a lot-sizing problem. Lot-sizing arises in production planning whenever changeover operations such as preheating, tool changing or cleaning are required between production runs of different products on a machine. The amount of the related changeover costs usually does not depend on the number of products processed after the changeover. Thus, to minimize changeover costs, production should be run using large lot sizes. However, this generates inventory holding costs as the production cannot be synchronized with the actual demand pattern: products must be held in inventory between the time they are produced and the time they are used to satisfy customer demand. The objective of lot-sizing is thus to reach the best possible trade-off between changeover and inventory holding costs while taking into account both the customer demand satisfaction and the technical limitations of the production system.

An early attempt at modelling this trade-off can be found in [19] for the problem of planning production for a single product on a single resource with an unlimited production capacity. Since this seminal work, a large part of the research on lot-sizing problems has focused on modelling operational aspects in more detail to answer the growing industry need to solve more realistic and complex production planning problems. An overview of recent developments in the field of modelling industrial extensions of lot-sizing problems is provided in [11].

In the present paper, we focus on one of the variants of lot-sizing problems mentioned in [11], namely the multi-product single-resource discrete lot-sizing and scheduling problem or DLSP. As defined in [7,11], several key assumptions are used in the DLSP to model the production planning problem:

- A set of products is to be produced on a single capacitated production resource.
- A finite time horizon subdivided into discrete periods is used to plan production.
- Demand for products is time-varying (i.e. dynamic) and deterministically known.
- At most one product can be produced per period (small bucket model) and the facility processes either one product at full capacity or is completely idle (discrete or all-or-nothing production policy).
- Costs to be minimized are the inventory holding costs and the changeover costs.

In the DLSP, it is assumed that a changeover between two production runs for different products results in a changeover cost. Changeover costs can depend either on the next product only (sequence-independent case) or on the sequence of products (sequence-dependent case). We consider in the present paper the DLSP with sequence-dependent changeover costs (denoted DLSPSD in the sequel). Sequence-dependent changeover costs are mentioned in

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[11] as one of the relevant operational aspects to be incorporated into lot-sizing models. Moreover, a significant number of real-life lot-sizing problems involving sequence-dependent changeover costs have been recently reported in the academic literature: see for instance [4] for an injection moulding process, [17] for a textile fibre industry or [6] for soft drink production.

A wide variety of solution techniques from the Operations Research field has been proposed to solve lot-sizing problems: the reader is referred to [3,10] for recent reviews on the corresponding literature. The present paper belongs to the line of research dealing with exact solution approaches, i.e. aiming at providing guaranteed optimal solutions to the problem. A large amount of existing solution techniques in this area consists in formulating the problem as a mixed-integer linear program (MILP) and in relying on a Branch & Bound type procedure to solve the obtained MILP. However the computational efficiency of such a procedure strongly depends on the quality of the lower bounds used to evaluate the nodes of the search tree. In the present paper, we seek to improve the quality of these lower bounds so as to decrease the total computation time needed to obtain guaranteed optimal solutions.

Within the last 30 years, much research has been devoted to the polyhedral study of lot-sizing problems in order to obtain the tight linear relaxations and improve the corresponding lower bounds: see e.g. [15] for a general overview of the related literature. In particular, valid inequalities which reduce the volume of the linear relaxation solution space by cutting off irrelevant parts have been proposed for several variants. Inequalities to strengthen the Capacitated Lot Sizing Problem (CLSP) are thus proposed in [1,13,14]. Contributions focusing specifically on the Discrete Lot Sizing Problem (DLSP) can be found in [2,5,8,18]. However, the known inequalities mainly exploit the underlying single-product subproblems and thus fail at *capturing the conflicts between multiple products sharing the same resource capacity*. This leads in some cases to significant residual integrality gaps for multi-product instances. In the present paper, we propose a new family of multi-product multi-period inequalities which enables us to partially remedy this difficulty for the DLSPSD. We then discuss both an exact and a heuristic algorithm to solve the corresponding separation problem. To the best of our knowledge, this is one of the first attempts at proposing multi-product valid inequalities for discrete lot-sizing problems.

The main contributions of the present paper are thus twofold. First we introduce a new family of valid inequalities representing conflicts on multi-period time intervals between several products simultaneously requiring production on the available resource. Second we formulate the corresponding separation problem as a quadratic binary program and propose to solve it either exactly by relying on a quadratic programming solver or approximately through a variable depth search heuristic algorithm of Kernighan–Lin type (see [12]). The results of our computational results show that the proposed inequalities are efficient at strengthening the linear relaxation of the problem and at decreasing the overall computation time needed to obtain guaranteed optimal solutions of the DLSPSD.

The remainder of the paper is organized as follows. In Section 2, we recall the initial MILP formulation of the multi-product DSLPSD as well as the previously published inequalities for the underlying single-product subproblems. We then present in Section 3 the proposed multi-product inequalities and discuss in Section 4 both an exact and a heuristic algorithm to solve the corresponding separation problem. Computational results are provided in Section 5.

2. MILP formulation of the DLSPSD

In this section, we first recall the initial MILP formulation of the DLSPSD. We use the network flow representation of changeovers between products, which was discussed among others in [2], as this leads to a tighter linear relaxation of the problem. We then present the inequalities proposed in [18] to strengthen the underlying single-product subproblems.

2.1. Initial MILP formulation

We wish to plan production for a set of products denoted $p=1\ldots P$ to be processed on a single production machine over a planning horizon involving T periods indexed $t=1\ldots T$. Product $p=0$ represents the idle state of the machine and period $t=0$ is used to describe the initial state of the production system.

Production capacity is assumed to be constant throughout the planning horizon. We can thus w.l.o.g. normalize the production capacity to one unit per period and apply a pretreatment on the original demand matrix resulting in a demand matrix containing only binary numbers (see [2,7,9]). We denote d_{pt} the demand for product p in period t: $d_{pt}=1$ in case there is a demand for product p in period t corresponding to producing p at full capacity in a period, $d_{pt}=0$ otherwise. Furthermore, we denote h_p the inventory holding cost per unit per period for product p and S_{pq} the sequence-dependent changeover cost to be incurred whenever the resource setup state is changed from product p to product q.

Using this notation, the DLSPSD can be seen as the problem of assigning at most one product to each period of the planning horizon while ensuring demand satisfaction and minimizing both inventory and changeover costs. We thus introduce the following binary decision variables:

- y_{pt} where $y_{pt}=1$ if product p is assigned to period t, 0 otherwise.
- w_{pqt} where $w_{pqt}=1$ if there is a changeover from product p to product q at the beginning of t, 0 otherwise.

This leads to the following MILP formulation denoted DLSPSD0 for the problem:

$$Z^* = \min \sum_{p=1}^{P}\sum_{t=1}^{T} h_p \sum_{\tau=1}^{t}(y_{p\tau}-d_{p\tau}) + \sum_{p,q=0}^{P} S_{p,q}\sum_{t=1}^{T-1} w_{p,q,t} \tag{1}$$

$$\sum_{\tau=1}^{t} y_{p\tau} \geq \sum_{\tau=1}^{t} d_{p\tau} \quad \forall p, \forall t \tag{2}$$

$$\sum_{p=0}^{P} y_{pt} = 1 \quad \forall t \tag{3}$$

$$y_{p,t} = \sum_{q=0}^{P} w_{q,p,t} \quad \forall p, \forall t \tag{4}$$

$$y_{p,t} = \sum_{q=0}^{P} w_{p,q,t+1} \quad \forall p, \forall t \tag{5}$$

$$y_{pt} \in \{0,1\} \quad \forall p, \forall t \tag{6}$$

$$w_{p,q,t} \in \{0,1\} \quad \forall p, \forall q, \forall t \tag{7}$$

The objective function (1) corresponds to the minimization of the inventory holding and changeover costs over the planning horizon. $\sum_{\tau=1}^{t}(y_{p\tau}-d_{p\tau})$ is the inventory level of product p at the end of period t. Constraints (2) impose that the cumulated demand over interval $[1,t]$ is satisfied by the cumulated production over the same time interval. Constraints (3) ensure that, in each period, the resource is either producing a single product or idle. Constraints (4) and (5) link setup variables y_{pt} with changeover