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A framework for and empirical study of algorithms for traffic assignment

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ABSTRACT

Traffic congestion is an issue in most cities worldwide. Transportation engineers and urban planners develop various traffic management projects in order to solve this issue. One way to evaluate such projects is traffic assignment (TA). The goal of TA is to predict the behaviour of road users for a given period of time (morning and evening peaks, for example). Once such a model is created, it can be used to analyse the usage of a road network and to predict the impact of implementing a potential project. The most commonly used TA model is known as user equilibrium, which is based on the assumption that all drivers minimise their travel time or generalised cost. In this study, we consider the static deterministic user equilibrium TA model.

The constant growth of road networks and the need of highly precise solutions (required for select link analysis, network design, etc.) motivate researchers to propose numerous methods to solve this problem. Our study aims to provide a recommendation on what methods are more suitable depending on available computational resources, time and requirements on the solution. In order to achieve this goal, we implement a flexible software framework that maximises the usage of common code and, hence, ensures comparison of algorithms on common ground. In order to identify similarities and differences of the methods, we analyse groups of algorithms that are based on common principles. In addition, we implement and compare several different methods for solving sub-problems and discuss issues related to accumulated numerical errors that might occur when highly accurate solutions are required.

1. Introduction and motivation

Because of the fast development of cities and road networks, transportation engineers face various difficulties of maintaining current roads and planning improvements and changes of the infrastructure in order to satisfy the existing and future travel demand. Since any changes to the urban infrastructure require large investments of time and money, making wise strategic decisions is very important [1]. One way to evaluate potential projects and analyse the usage of existing road networks is to apply mathematical modelling. As presented in Ortúzar and Willumsen [1], transportation planning tools include many different mathematical models designed to solve various tasks. In our study, we concentrate on the traffic assignment (TA) problem that is part of transportation planning.

The TA model describes travel behaviour of road users. In particular, this model is designed to predict what route choice every individual will make during a given period of time. The conventional approach to model the behaviour of travellers is to make some assumptions on how people choose routes and to find a traffic flow pattern satisfying these assumptions. The most well-known assumptions are the ones following Wardrop's first principle (also called user equilibrium condition): "*The journey times on all the routes actually used are equal, and less than those which would be experienced by a single vehicle on any unused route*" [2].

This principle models the behaviour of travellers by assuming that all drivers are selfish and that they choose the fastest routes going from their origin to their destination. As a result, an *equilibrium state* is achieved, when no one has an incentive to switch to another route. This principle allows different mathematical formulations of the problem based on different additional assumptions. The classical model that is commonly used in practice is a static deterministic traffic assignment (TA) that is the subject of this paper. Other traffic assignment problems such as the TA problem with elastic demand (see [3]) or dynamic TA (see [4]) are not considered in this paper.

This classical model was developed in the 1950s and since then various algorithms have been proposed to solve it. The wide research interest in this problem has several reasons. First, TA is a

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challenging problem that arises in different practical applications. Second, the existing transportation models continue to grow and become more detailed. For example, in 2006 the ART model (Auckland Regional Transport Model) included 202 zones, whereas in 2008 it contained 512 zones [5]. A *zone* refers to an area of a transportation network that can vary from a city block to a neighbourhood [6]. Third, high accuracy of the solution is required for select link analysis[1] and for consistent comparison between design scenarios, as presented in Slavin et al. [7] and Gentile [8]. Therefore, we can conclude that there is a growing need for efficient algorithms able to solve TA problems of realistic size with high accuracy.

Regardless the number of proposed algorithms in the literature, there is no comprehensive survey study comparing them. To the best of our knowledge, only the paper of Inoue and Maruyama [9] analyses many different methods under the same computational environment. The authors implemented 11 algorithms for traffic assignment. However, all implementation details are omitted in the paper. It is not clear if the authors used the same framework for all algorithms and how carefully they followed the descriptions of the algorithms available in the literature.

The aim of our research is to analyse and compare the most promising approaches for solving TA in the context of a *framework* that can be shared by different algorithms. We compare methods without considering any special implementation details that may have been used in commercial or research software. Instead, we focus on the use of common code wherever possible. This will allow testing general ideas of the methods without giving an advantage to any of them.

Another motivation of this study is to identify the advantages and disadvantages of different *groups* of algorithms (the classification is presented in Section 3). Early on only link-based approaches were available because of memory limitations. In recent years, however, other types of algorithms have been implemented in many commercial software packages used by practitioners. One of the main advantages of new algorithms is that they can provide the path choice information which is necessary to evaluate effects of schemes such as congestion pricing without re-running the algorithm [10]. Therefore, it becomes important to analyse them and to compare them with classical link-based approaches. Other factors that motivate our research are highlighted in Section 3 along with a literature overview.

Preliminary results of our study can be found in Perederieieva et al. [11].

The rest of the paper is organised as follows. Section 2 states the deterministic static traffic assignment problem. Section 3 is devoted to a literature review and our choice of algorithms for comparison. In Section 4, various implemented methods for solving traffic assignment are described. Section 5 discusses the computational study and comparison of algorithms. Finally, Section 6 presents conclusions and future work.

2. Problem formulation

This section introduces a mathematical formulation of the TA problem and notation that is used throughout the paper.

A transportation network is defined as a directed graph $G(N, A)$ where N is a set of nodes and A is a set of links. The users of the transportation network travel from their origins to their destinations. Let D_p denote travel demand between origin–destination (O–D) pair $p \in Z$, where Z is the set of all O–D pairs. A *demand* represents how many vehicles are travelling from an origin to a destination.

The key feature of TA models consists in taking into account congestion effects that occur in road networks. In order to consider congestion, link cost functions are introduced into the model. They represent travel times through links of a network depending on the traffic flow on those links. Let $c_a(\mathbf{f})$ denote a link cost function of link a that depends on link flows $\mathbf{f} = (f_1, f_2, \ldots, f_{|A|})$. Link flow is the number of vehicles per time unit on each link.

Let $\mathbf{F} = (F_1, \ldots, F_{|K|})$ denote a vector of path flows, where K is the set of all simple paths of graph $G(N, A)$. Path flows are related to link flows by the following expression:

$$f_a = \sum_{p \in Z} \sum_{k \in K_p} \delta_a^k F_k, \tag{1}$$

where δ_a^k equals one if link a belongs to path k, and zero otherwise; $K_p \subseteq K$ is the set of paths between O–D pair p. Let path cost function $C_k(\mathbf{F})$ denote the travel time on path k.

The conventional model of the traffic assignment problem is based on two assumptions that allow to formulate and solve it as a mathematical programme.

1. *Additivity* of path cost functions: travel time on each path is the sum of travel times of links belonging to this path, i.e. $C_k(\mathbf{F}) = \sum_{a \in A} \delta_a^k c_a(\mathbf{f})$.
2. *Separability* of link cost functions: travel time on each link depends only on flow on this link, i.e. $c_a(\mathbf{f}) = c_a(f_a)$.

If these assumptions are satisfied, solving the following optimisation problem (2) results in the link flows satisfying the user equilibrium condition [6]:

$$\begin{aligned} &\min \sum_{a \in A} \int_0^{f_a} c_a(x)\, dx \\ &\sum_{k \in K_p} F_k = D_p, \quad \forall p \in Z, \\ &F_k \ge 0, \quad \forall k \in K_p, \forall p \in Z, \\ &f_a = \sum_{p \in Z} \sum_{k \in K_p} \delta_a^k F_k, \quad \forall a \in A. \end{aligned} \tag{2}$$

If all path cost functions $C_k(\mathbf{F})$ are *positive and continuous* then existence of a solution of TA is ensured. If, furthermore, the path cost functions are *strictly monotone*, the solution is guaranteed to be unique [12]. In the following, it is assumed that these requirements are satisfied.

The formulation (2) is sometimes referred to as link-route or path flow formulation [13,14]. We will use the latter term. The path-based algorithms that are discussed in Section 4.2 use this mathematical programme. Other optimisation formulations of the TA problem based on a different set of decision variables can be found in Patriksson [13] and Bertsekas [14].

3. Literature overview

One of the possible ways to classify traffic assignment algorithms is according to how the solution is represented: link-based (solution variables are link flows), path-based (solution variables are path flows) and bush-based (solution variables are link flows coming from a particular origin), see [15].

Historically, the first algorithms developed for solving the traffic assignment problem were link-based. The most well-known such algorithm is the Frank–Wolfe (FW), a general algorithm for convex optimisation problems [16]. Due to its simplicity and low memory requirements, it is used even now and is implemented in different commercial software packages. However, this algorithm is known to tail badly in the vicinity of the optimum and usually cannot be

[1] Select link analysis provides information of where traffic is coming from and going to for vehicles at selected links (and combination of links) throughout the modelled network [1].