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Fleet-sizing for multi-depot and periodic vehicle routing problems using a modular heuristic algorithm

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ABSTRACT

In this paper, we address the problem of determining the optimal fleet size for three vehicle routing problems, i.e., multi-depot VRP, periodic VRP and multi-depot periodic VRP. In each of these problems, we consider three kinds of constraints that are often found in reality, i.e., vehicle capacity, route duration and budget constraints. To tackle the problems, we propose a new Modular Heuristic Algorithm (MHA) whose exploration and exploitation strategies enable the algorithm to produce promising results. Extensive computational experiments show that MHA performs impressively well, in terms of solution quality and computational time, for the three problem classes.

1. Introduction

In the classical vehicle routing problem (VRP), a homogeneous fleet of vehicles services a set of customers from a single distribution depot or terminal. Each vehicle has a fixed capacity that cannot be exceeded and each customer has a known demand that must be fully satisfied. Each customer must be serviced by exactly one visit of a single vehicle and each vehicle must depart from the depot and return to the depot [1].

Several variations and specializations of the VRP, ranging from the Capacitated VRP (CVRP) to more complex problems including various realistic attributes and constraints, have been extensively studied in the past five decades. However, the literature underlines that there still exist many VRP variants that have not received adequate attention despite their important relevance to many applications.

Among such VRP variants, the Multi-Depot Periodic VRP (MDPVRP) is the one which has not been studied too much, relative to the other existing variants, despite the fact that it reflects many real-life applications (e.g., food collection and distribution [2], maintenance operation [3], and raw material supply [4]). In this paper, we contribute toward addressing this challenge by studying a variant of the MDPVRP, which we refer to as the fleet-size Multi-Depot Periodic VRP (fs-MDPVRP), and two of its special cases, the fleet-size Periodic VRP (fs-PVRP) and the fleet-size Multi-Depot VRP (fs-MDVRP).

The fs-MDPVRP studied in this paper, contrary to the classical MDPVRP in which the goal is often to minimize the total travel distance (cost), seeks to determine the optimal number of vehicles needed for delivery operations over a given planning horizon. However, the fs-MDPVRP shares the network structure and several characteristics with the classical MDPVRP. More precisely, in this VRP, it is assumed that there exists a finite number of depots where vehicles are located. Each vehicle performs only one route per period and each vehicle route must start and finish at the same depot. Each customer may require to be visited on different periods during the planning horizon and these visits may only occur in one of a given number of allowable visit-period combinations. In this problem, as mentioned, the goal is to determine the optimal fleet size where three practical constraints, i.e., vehicle capacity, maximum route duration and budget constraints, should be satisfied. A fs-MDPVRP incorporating such characteristics reduces to a fs-PVRP or a fs-MDVRP if the number of depots or periods is set to 1.

To the best of our knowledge, there is no significant contribution in the literature to address the above VRPs. In this paper, to tackle each of the considered problems, we propose a heuristic solution method that builds a solution at each iteration using a procedure that has three phases, each focusing on one of the decisions to be made (the selection of periods when each customer will be served, the assignment of customers to depots, and the design of routes). The proposed heuristic algorithm incorporates different exploration and exploitation strategies to produce good results, in terms of solution

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quality and computational efficiency. The proposed heuristic solution method is numerically shown to be efficient to address a wide variety of test problems for the three problem classes.

The remainder of this paper is organized as follows: Section 2 gives the problem statement. In Section 3, the literature survey relevant to the topic of this study is presented. Different aspects of the proposed heuristic algorithm are described in Section 4. The experimental results are given in Section 5. Finally, Section 6 provides conclusions and the evaluation of the work.

2. Problem statement and modeling

In this section, we formally state each of the problem classes, introducing the notations used throughout this paper.

fs-MDVRP: Consider an undirected graph $G(V,E)$. The node set V is the union of two subsets $V = V_C \cup V_D$, where $V_C = \{C_1, \dots, C_N\}$ represents the customers and $V_D = \{D_1, \dots, D_M\}$ includes the depots. With each node $i \in V_C$ is associated a deterministic demand q_i. The edge set E contains an edge for each pair of customers and for each depot-customer combination. There are no edges between depots. With each edge $(v_i, v_j) \in E$ is associated a travel distance d_{ij} [5].

fs-PVRP: In the fs-PVRP, the undirected graph $G(V,E)$ is modified by fixing the value of M to one and by introducing a planning horizon of T periods. In such a graph, each customer i is characterized by a service frequency f_i, stating how often within these T periods the customer must be visited and a list L_i of possible visit-period combinations [6].

fs-MDPVRP: Finally, the fs-MDPVRP combines the two above problem settings, asking for the selection of a depot and a visit pattern for each customer, with services in different periods to the same customer being required to originate at the same depot [6].

In each problem class, the goal is to minimize the maximum number of vehicles used over the planning horizon. By considering such an objective function, we actually address the strategic version of the problem in which we assume that vehicles are either bought or rented over a long period of time by the company facing the problem.

To model the above problems, we consider the three following restrictions which reflect important requirements that are often found in real-life applications:

1. *The vehicle capacity constraint*: This constraint states that the total demand of the customers on any route should not exceed the vehicle capacity Q.
2. *The route duration constraint*: This constraint ensures that the total duration of a route does not exceed a preset value D.
3. *The budget constraint*: In many logistical systems, one is usually faced with budgetary constraints that come from the fact that a limited investment budget is available for a certain area or a certain period of time. Budget considerations are almost always ignored when dealing with VRPs. In this paper, we consider such a budgetary restriction which we refer to as the Travel-Distance Budget (TDB) constraint. The TDB constraint reflects many real-life cases (for example, garbage collection and milk distribution systems) in which, due to limited financial resources to completely cover system's operating costs (e.g., high fuel price and vehicle depreciation costs), vehicles cannot be practically allowed to travel more than a prespecified distance. In this study, the TDB constraint is defined using two different models, each realizing an important managerial challenge in real-life distribution and logistical systems. In the first model (R_1), we set a bound on the total distance that vehicles are permitted to travel over the planning horizon. On the other hand, the second model (R_2) aims to reflect the situations in which, due to geographical and operational constraints, the total distance traveled by vehicles assigned to a depot cannot exceed an imposed limit in each period.

Depending on how we model the TDB constraint, each of the above problem classes can be expressed by one of the following mathematical programming models:

$$(R_1)\ \min K \tag{1}$$

subject to

$$F(x,K) \le b \tag{2}$$

$$\tau \le \epsilon \tag{3}$$

where K is the maximum required number of vehicles (fleet size) needed over the planning horizon. More precisely, K is defined as the sum of the vehicles needed at each depot. Constraint (2) corresponds to the vehicle-capacity and route-duration restrictions described above. Constraint (3) imposes that the total traveled distance (τ) is limited by a positive value ϵ:

$$(R_2)\ \min K \tag{4}$$

subject to

$$F(x,K) \le b \tag{5}$$

$$\tau_{tj} \le \epsilon_{tj} \quad \forall t \in T, \quad \forall j \in D; \tag{6}$$

where τ_{tj} is the total distance traveled by the vehicles assigned to depot j in period t and ϵ_{tj} is a positive upper bound which is set on τ_{tj}.

Ref. [5] showed that the formulation of a generalized PVRP includes the MDVRP as a special case by associating a different period to each depot, such that the ith customer has a frequency $f_i = 1$ and can be visited in any period. Ref. [6] extended this result by proving that a MDPVRP with T periods and D depots can be transformed into a generalized PVRP by associating a period to each (period, depot) pair, such that the ith customer, having a list L_i of patterns, is visited f_i times over the planning horizon using one of the $D \times L$ patterns. We rely on these two transformations in the development of the proposed modular heuristic algorithm.

3. Literature review

In this section, we focus on reviewing papers formerly published in the literature to address different settings of the MDPVRP and its two special cases, i.e., the PVRP and the MDVRP. The goal of this review is first to present the most recently proposed heuristic and meta-heuristic algorithms for these VRPs, and to highlight that there is no significant contribution dealing with the problem settings considered in this paper.

To the best of our knowledge, all the MDVRPs and PVRPs studied in the literature consider the total travel distance (cost) as the main goal, regardless of constraint types that they deal with. The majority of solution methods, put forward to address these problems, are divided into (1) classical heuristics which range from simple construction and improvement procedures [7–11] to more structured algorithms as iterative heuristics [12–14] and multi-phase solution methods [15–17], and (2) meta-heuristics which clearly outperform the classical heuristics by benefiting of better exploitation and exploration strategies. In a general point of view, these methods are grouped into tabu search algorithms [18,5], variable neighbourhood search [19,20], large-scale neighbourhood search algorithm [21], and evolutionary meta-heuristics as genetic algorithm [22,6], ant colony optimization [23], and scatter search [4].

Similar to the above problem settings, all the MDPVRPs existing in the literature address the problem of minimizing the total travel distance (cost). Solution methods, targeting these MDPVRPs, are divided into two main groups: (1) classical heuristics, which often address the problem in a sequential manner, and (2) meta-heuristics