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A penalty search algorithm for the obstacle neutralization problem

Ali Fuat Alkaya [a,*], Vural Aksakalli [b], Carey E. Priebe [c]

[a] Computer Engineering Department, Marmara University, Istanbul 34722, Turkey
[b] Industrial Engineering Department, Istanbul Sehir University, Istanbul 34662, Turkey
[c] Applied Mathematics & Statistics Department, Johns Hopkins University, MD 21218, USA

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ABSTRACT

We consider a path planning problem wherein an agent needs to swiftly navigate from a source to a destination through an arrangement of obstacles in the plane. We suppose the agent has a limited neutralization capability in the sense that it can safely pass through an obstacle upon neutralization at a cost added to the traversal length. The agent's goal is to find the sequence of obstacles to be neutralized en route that minimizes the overall traversal length subject to the neutralization limit. We call this problem the obstacle neutralization problem (ONP), which is essentially a variant of the intractable weight-constrained shortest path problem in the literature. In this study, we propose a simple, yet efficient and effective suboptimal algorithm for ONP based on the idea of penalty search and we present special cases where our algorithm is provably optimal. Computational experiments involving both real and synthetic naval minefield data are also presented.

1. Introduction

The problem of finding shortest paths in the presence of a resource constraint is observed in many real life situations. Several such cases are described below:

- The problem of minimizing the time for data to reach destination subject to a given total delay limit in the telecommunications industry [1],
- the problem of finding the path for a military aircraft with minimum probability of being detected by enemy radar subject to fuel constraints [2], and
- the problem of approximating a curve with maximum number of breakpoints subject to storage constraints in computer graphics [3].

Problems such as above are known as the weight-constrained shortest path problem (WCSPP) in the literature, which is NP-Hard except for some trivial cases [4]. In this work, we study a variant of this problem in a path planning setting. Specifically, we consider a problem where the goal is to safely and swiftly navigate an agent from a given source location to a destination through an arrangement of disk-shaped obstacles in the plane. We assume the agent has a neutralization capability in the sense that it can safely pass through an obstacle upon neutralization at a cost added to the traversal length. However, we make the restriction that the number of allowed neutralizations is limited, perhaps due to payload capacity of the agent. The agent's goal is to find the optimal sequence of obstacles to be neutralized en route that minimizes the overall traversal length subject to the number of neutralizations available. We call this problem the obstacle neutralization problem (ONP). It should be noted if the number of allowed neutralizations is unlimited, then the problem reduces to the classical unconstrained shortest path problem and can be solved in polynomial time. Hence, we limit our focus in this study on ONP instances where the optimal solution requires utilization of the entire neutralization capability.

ONP can be defined formally as follows: Let

- the obstacle field Ω be a finite region in $\mathbb{R}^2$,
- the starting location s and the destination t be two points in Ω,
- the obstacle set D be a finite set of disks in Ω with radius r,
- the neutralization cost be denoted by $C \geq 0$, and
- the neutralization limit be denoted by $K \leq |D|$.

An agent needs to traverse from s to t along a continuous curve which is as short as possible in the sense of arclength. The agent cannot enter the obstacle disks but, if and when the agent is located at the boundary of a disk $d_i \in D$, the agent has the option to neutralize it and pass through at a cost C added to the traversal length. The agent's goal is to find a sequence of at most K obstacles

* Corresponding author.
E-mail addresses: falkaya@marmara.edu.tr (A.F. Alkaya), aksakalli@sehir.edu.tr (V. Aksakalli), cep@jhu.edu (C.E. Priebe).

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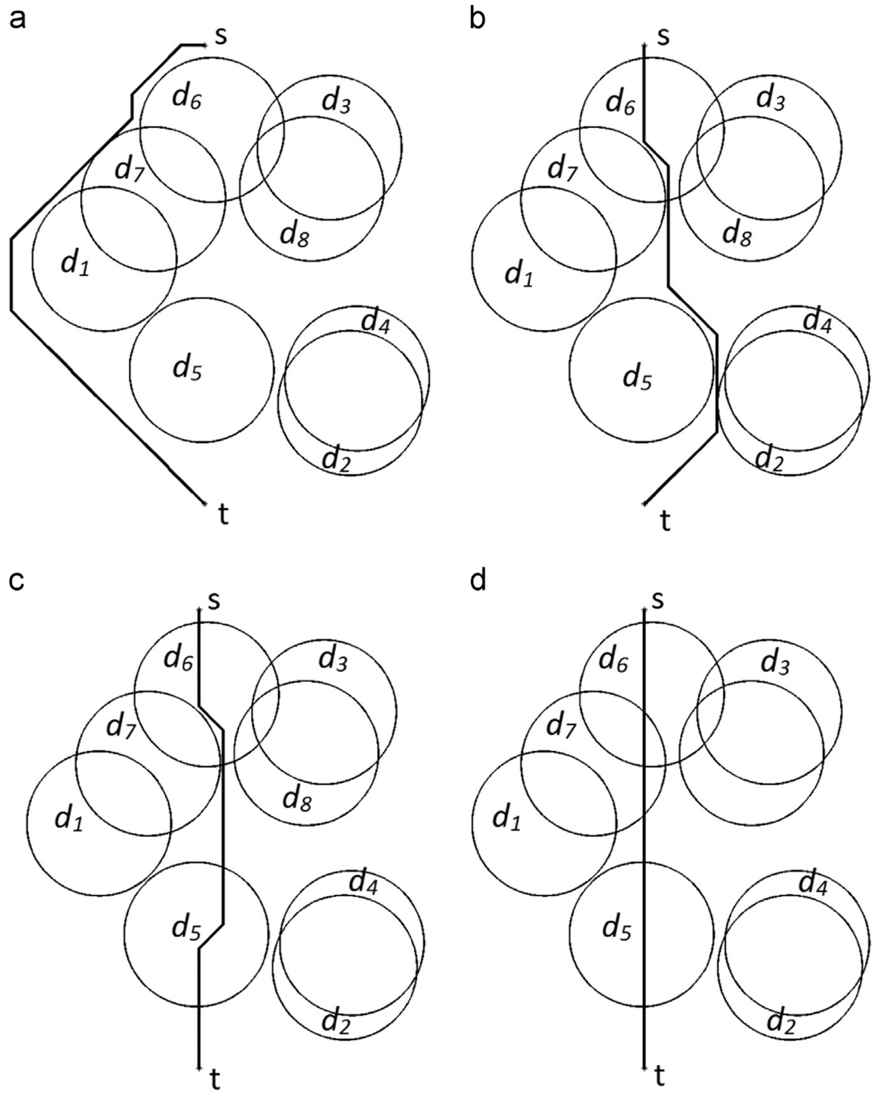

Fig. 1. An instance of ONP and the associated optimal paths for $K=0$, 1, 2, and 3 respectively.

to be neutralized en route that minimizes the overall s–t traversal length. Throughout this study, we assume that all disks have the same neutralization cost and the same radius. For simplicity and convenience, an instance of ONP will be denoted by the tuple (s, t, D, C, K).

ONP is a combinatorial optimization problem where among $|D|$ disks, the agent must find the optimal permutation of at most K of the disks for neutralization. Clearly, ONP is a variant of the WCSPP where the weight constraint is the number of neutralizations available. On the other hand, to our knowledge, ONP as defined above has not been studied before in the open literature except for a brief mention in [5].

An instance of ONP is shown in Fig. 1 with $s=(10,20)$, $t=(10,1)$, and $C=0.8$ with $r=3$. In the figure, the optimal paths for $K=0$, 1, 2 and 3 are given in subfigures 1(a), (b), (c), and (d), respectively. Note that for $K=1$, only d_6 is neutralized; for $K=2$, both d_5 and d_6 are neutralized; and for $K=3$, d_5, d_6, and d_7 are neutralized. Note also that it is not always the case that all K allowable neutralizations are used.

Our goal in this study is to present a simple, fast, efficient, and scalable algorithm for ONP which we call penalty search algorithm (PSA). This algorithm is especially suitable for online applications. State-of-the-art algorithms proposed for problems closely related to ONP either require significant run times or show poor performance on ONP instances whereas PSA finds optimal or near-optimal results in short execution times. In particular, we present computational experiments comparing the performance of PSA to two other algorithms on both real and synthetic naval minefield sample data: (1) adaptation of another popular suboptimal WCSPP algorithm to ONP, called Delay-Constrained Unicast Routing (DCUR) Algorithm, and (2) an exact algorithm for ONP. Our results suggest solutions found by PSA compare very favorably to those obtained by the exact algorithm while requiring substantially less computational resources whereas DCUR exhibits relatively poor performance in general.

The remainder of this manuscript is organized as follows: Section 2 relates ONP to WCSPP and gives a comprehensive overview of current state-of-the-art in WCSPP research. Section 3 provides details of PSA, discusses its properties, and proves its optimality in certain cases. Section 4 presents our computational experiments, and Section 5 summarizes and concludes our work.

2. WCSPP and previous work

2.1. WCSPP and ONP

For a given graph $G=(V,E)$, let τ_{ij} and θ_{ij} denote the cost and weight of the edge $(i,j)\in E$ respectively. The objective in WCSPP is